UNIVERSIDAD DE GUANAJUATO

Universidad de Guanajuato

División de Ciencias e Ingenierías

Massive Neutrinos: A Scotogenic Extension with Gauged Lepton Number

Master thesis presented by

José Andrés Enríquez López

A thesis submitted in fulfillment of the requirements
to obtain the degree of Master in

Physics

Supervisor: **Dr. Carlos Alberto Vaquera Araujo**

December 2023

Declaration

I hereby declare that this master's thesis is the result of my own original research and authorship. I have duly acknowledged and cited all the sources used in this thesis in the appropriate manner.

Furthermore, I affirm that all the work presented herein is the outcome of my individual efforts during my graduate studies at the University of Guanajuato, except where specifically indicated and referenced.

Acknowledgments

I express my deepest gratitude to my mother, whose constant support, encouragement, and care have sustained me throughout the years. Despite our occasional differences, she has consistently stood by my side. I also extend my thanks to my siblings, whose presence has enriched my life with joy and fun. Their influence drives me to continually strive to be the person I aspire to become and to make them proud. I am equally thankful to the rest of my family for their high expectations, and I also want to thank Tobi, whose presence has brought light to my home over the past two years.

A special acknowledgment goes to my advisor, Carlos Vaquera, for his unwavering support, guidance, and knowledge-sharing during our months of collaboration. His insights and support have been invaluable, and without his mentorship, this work would not have been possible.

I extend my gratitude to my classmates, who have been companions on our shared journey at the University, creating memorable moments over the past two years.

I would like to recognize CONACYT for the financial support that has played a crucial role in enabling not only my academic pursuits but also those of many graduate students pursuing their dreams.

Lastly, I want to express my appreciation to all my friends who have believed in me and contributed to my personal growth. Whether we remain in touch or not, each of them has brought something uniquely special to my life. To every individual who has been part of my journey, thank you.

Abstract

This thesis addresses two significant aspects of contemporary physics: the generation of neutrino mass and the study of dark matter. Despite being initially considered massless in the Standard Model of particle physics, experimental evidence now confirms that neutrinos do possess mass. In this work, we examine popular mechanisms explaining neutrino mass generation and present a model that not only provides a mechanism to origin neutrino mass but also introduces dark matter candidates. Following an introduction to Group Theory and the Standard Model, we explore mechanisms for neutrino mass generation and review key concepts related to dark matter. The scotogenic model is presented and serves as the foundation of our theoretical framework, extending the Standard Model by elevating lepton number to a gauge symmetry. This additional symmetry necessitates the introduction of extra particles, and we describe their interactions and resulting phenomenology. The results of this thesis will be extended in a future publication. By exploring the connections between neutrino masses and dark matter within models beyond the Standard Model, we aim to deepen our understanding of fundamental particles and their role in shaping the universe.

Contents

1

Introduction

The Standard Model (SM) [1–3] is widely regarded as one of the most successful physical theories due to its remarkable ability to explain a wide range of particle physics phenomena. It encompasses the fundamental particles and the electromagnetic, strong, and weak interactions. However, despite its accomplishments, the SM has certain limitations, particularly in the context of massive neutrinos [4–6] and dark matter [7–9].

Neutrinos were first proposed by W. Pauli in 1930 to account for the continuous beta decay emission spectrum [10, 11]. Pauli proposed that neutrinos possessed mass and were electrically neutral, but subsequent work by E. Fermi [12], who popularized the term "neutrino," and F. Perrin [13] suggested that they were actually massless. In addition, in 1934 Bethe and Peierls [14] showed that the cross-section between a neutrino and a proton should be extremely small and theorized that neutrinos would never be observed. Within the framework of the Standard Model, neutrinos are treated as massless entities. However, experimental evidence from neutrino oscillation experiments [15–17] indicates that neutrinos must have mass. This inconsistency points to the incompleteness of the Standard Model and the need for new physics to elucidate the origin and remarkably small magnitude of neutrino mass compared to other fermions in the Standard Model.

Exploring various mechanisms to confer mass to neutrinos is crucial for advancing towards a more comprehensive theory of particle physics, ensuring consistency with existing experimental constraints [18]. Furthermore, understanding neutrino mass may reveal connections to other unresolved questions in particle physics, such as the nature of dark matter [19].

Dark matter, constituting approximately 27% of the universe's energy according to cosmological observations, is a form of matter that interacts weakly with Standard Model particles. The Standard Model lacks a viable candidate for dark matter, explaining only about 4% of the total energy content of the universe [20]. Consequently, the existence of a new type of matter, likely in the form of a fundamental particle beyond the Standard Model, is required to account for dark matter. Such particle candidates should be stable over cosmological time scales, given that dark matter should have existed from the early universe until today.

An intriguing possibility is the existence of a common origin between neutrino mass and dark matter. For instance, dark matter could potentially serve as the mediator for neutrino mass generation [21–24], or the symmetry stabilizing dark matter might be intimately related to neutrinos [25, 26]. Understanding the nature of both neutrinos and dark matter holds the potential to illuminate unresolved issues at the interface of cosmology and particle physics.

In Chapter 2 of this thesis, we provide an overview of the theoretical foundations necessary to comprehend the subsequent content. This includes an introduction to Group Theory, the Standard Model of Particle Physics, and dark matter. Chapter 3 focuses on reviewing models capable of explaining the generation of massive neutrinos, such as the seesaw mechanism and the Scotogenic Model [27]. In Chapter 4, we present an extension to the Scotogenic Model, wherein lepton number is gauged and then spontaneously broken by three units, leading to light Dirac neutrino masses induced through a radiative mechanism and the appearance of dark matter candidates.

2

Theoretical foundations

Contents

In this introductory chapter, we cover background topics relevant to the subsequent chapters. The chapter is divided into three main sections.

In Section 2.1, we provide an introduction to group theory in the context of the Standard Model. We begin with a series of definitions and fundamental facts in Section 2.1.1. Following this, we focus on the relevant groups in the Standard Model, namely SU(2) and SU(3), and discuss their properties in Sections 2.1.2 to 2.1.4.

Moving on to Section 2.2, we review the fundamentals of the Standard Model. First, we present the particle content of the Standard Model in Section 2.2.1, detailing the various elementary particles and their properties. In Section 2.2.2, we delve into the interactions between these particles, encompassing the electromagnetic, weak, and strong forces.

Sections 2.2.3 and 2.2.4 are dedicated to understanding the mechanism responsible for giving mass to particles in the Standard Model. Here, we discuss the Higgs mechanism and spontaneous symmetry breaking, pivotal elements in generating masses for particles and breaking the electroweak symmetry.

In Section 2.2.6, we introduce the Lorentz group, a fundamental concept in relativistic physics. Section 2.3 is dedicated to dark matter, presenting the evidence for it and potential candidates.

2.1 Introduction to group theory

2.1.1 Lie Groups and Algebras

In this section, we present key concepts of group theory without formal proof. The following definitions and properties are drawn primarily from the books by Zee [28] and Keski-Vakkuri [29], which serve as references for the remaining sections of this chapter.

Consider a set $G = \{a, b, ...\}$ equipped with a binary composition law. We define a group as the pair $(G, \cdot)$, or simply G, which satisfies the following conditions:

1. Closure: For all $a, b \in G$, $a \cdot b \in G$.
2. Associativity: For all $a, b, c \in G$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
3. Identity: There exists an element e such that for all $a \in G$, $a \cdot e = e \cdot a = a$.
4. Inverse: For all $a \in G$, there exists an element a^{-1} such that $a \cdot a^{-1} = a^{-1} \cdot a = e$.

An Abelian group is a group that satisfies the additional commutativity condition: for every $a, b \in G$, $a \cdot b = b \cdot a$. If $a \neq b$, the group is called non-Abelian.

Now, we introduce some definitions that are useful in the context of group theory:

- Order: The order of a group refers to the number of elements in the group, often denoted as $|G|$.

- Subgroup: A group J is a subgroup of group G if every element in J is also in G, and the order of J is a factor of the order of G.

- Homomorphism: Given two sets X and Y, a mapping $f : X \to Y$ is a homomorphism when it preserves some structure. For example, if X and Y are groups, a homomorphism preserves the composition law of the groups. That is, if $x_1, x_2 \in X$ and $f(x_1), f(x_2) \in Y$, then $f(x_1)f(x_2) = f(x_1 x_2)$.

Beyond these fundamental characteristics, group theory encompasses a vast and profound realm of mathematical exploration. One relevant aspect within this field is the concept of a **group representation**[30], denoted by D. A group representation is a homomorphism that maps elements $g_i \in G$ of a group G to linear operators $D(g_i)$. Alternatively, a group representation can be understood as a one-to-one correspondence between elements of the group G and matrices, expressed as $D(g)$ for $g \in G$. A representation D is said to be faithful if it is injective.

The vector space on which these matrices, $D(g)$, operate is commonly referred to as the representation space. The dimension of the representation space corresponds to the dimension of the representation itself. If we consider a vector from the representation space of $D(g)$ and observe that the action of $D(g)$ on this vector, within a specific subspace, yields another vector residing in the same subspace, then the representation is categorized as reducible. Conversely, an irreducible representation does not possess such invariant subspaces. In summary, these properties enable us to express any reducible representation of the group G as a direct sum of irreducible representations. The groups that are relevant to us are the **Lie groups**. Lie groups are groups whose elements are labeled by a set of continuous parameters.

Another concept relevant to our purposes is the direct product. The direct product of two vectors u and v is denoted by (u, v). An operator $U : V \to V$ is considered unitary if, for every $v, w \in V$, $(v, w) = (Uv, Uw)$. A unitary representation of a group G is a homomorphism that maps elements of the group to unitary operators.

The orthogonal group of degree n is defined as the group of real matrices whose inverse coincides with their transpose:

$$\mathrm{O}(n, \mathbb{R}) = \left\{ A \in \mathrm{GL}(n, \mathbb{R}) : A^T A = A A^T = \mathbb{1}_n \right\}. \tag{2.1}$$

The subset of $\mathrm{O}(n, \mathbb{R})$ including matrices with determinant 1 is called the special orthogonal group of degree n:

$$\mathrm{SO}(n, \mathbb{R}) = \left\{ A \in \mathrm{GL}(n, \mathbb{R}) : A^T A = A A^T = \mathbb{1}_n, \det A = 1 \right\}. \tag{2.2}$$

The generalization of the orthogonal group of degree n to the complex field is the unitary group of degree n, defined as:

$$\mathrm{U}(n, \mathbb{C}) = \left\{ A \in \mathrm{GL}(n, \mathbb{C}) : A^\dagger A = A A^\dagger = \mathbb{1}_n \right\}, \tag{2.3}$$

where $A^\dagger$ is the conjugate transpose of A. The subset of $\mathrm{U}(n)$ containing matrices with determinant equal to 1 constitutes the special unitary group of degree n:

$$\mathrm{SU}(n) = \{ A \in \mathrm{U}(n), \det A = 1 \}. \tag{2.4}$$

Consider an element $\mathrm{U} \in \mathrm{SU}(n)$. U can be represented using the exponential map:

$$\mathrm{U} = e^{-i\bar{\theta}\cdot X} = e^{-i\theta_a X_a}, \tag{2.5}$$

where θ_a are real parameters, and X_a represents matrices. Each X_a can be found by computing:

$$X_a = i \left. \frac{\partial \mathrm{U}(\bar{\theta})}{\partial \theta_i} \right|_{\bar{\theta}=0}. \tag{2.6}$$

These matrices X_a are called the generators of the (Lie) group. The generators form a basis for the real Lie algebra associated with the corresponding Lie Group. With this, we can define a Lie algebra as a linear space spanned by linear combinations $\sum_i \theta_i X_i$ of the generators.
The commutator is defined as $[A, B] = AB - BA$. The Lie Algebra is defined by the commutation relations between the generators of the group. These commutation relations are written as:

$$[X_a, X_b] = i f_{abc} X_c, \tag{2.7}$$

where f_{abc} are called structure constants. For example, the Lie algebra $\mathfrak{so}(3)$ is defined by the commutation relations:

$$[J^i, J^j] = i\epsilon_{ijk} J^k. \tag{2.8}$$

These relations are fulfilled by the generators of SU(2):

$$J^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\sigma^1}{2}, \quad J^2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{\sigma^2}{2}, \quad J^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\sigma^3}{2}, \tag{2.9}$$

where we have introduced the Pauli matrices σ^i, which satisfy the following relations:

$$[\sigma^i, \sigma^j] = 2i\epsilon_{ijk}\sigma^k, \quad \{\sigma^i, \sigma^j\} = 2\delta_{ij}\mathbf{1}, \quad \sigma^i\sigma^j = \delta_{ij}\mathbf{1} + i\epsilon_{ijk}\sigma^k. \tag{2.10}$$

The group elements of SU(2) are of the form:

$$U = e^{-i\frac{\sigma^i}{2}\theta^i}, \tag{2.11}$$

which transform two-component objects called spinors:

$$\chi \to \chi' = U\chi, \tag{2.12}$$

leaving their Hermitian product invariant $\eta^\dagger\chi' = \eta^\dagger U^\dagger U\chi = \eta^\dagger\chi$.

The groups SO(3) and SU(2) are locally equivalent, but there is a global difference. For example, if we take a 2π rotation around the z axis, we can represent that rotation by:

$$e^{-2\pi i J_3} = \mathbb{1}_{3\times 3} \tag{2.13}$$

for vectors under SO(3). Nevertheless, the same transformation for spinors under SU(2) is:

$$e^{-2\pi i J^3} = -\mathbb{1}_{2\times 2}. \tag{2.14}$$

Thus, two complete rotations are needed to return to the original state.

2.1.2 Irreducible representations of SU(2)

The groups that are relevant for the Standard Model, and most of particle physics, are U(1), SU(2), and SU(3). In this section, we focus on the study of the SU(2) group.

It is common to define the su(2) algebra in terms of the generators J_a which satisfy the commutation relations:

$$[J_a, J_b] = i\epsilon_{abc}J_c. \tag{2.15}$$

Supposing that J_3 is represented by an $N\times N$ hermitian matrix and diagonalizing it, our objective is to determine the matrix representation for the elements of the rotation group and ascertain the dimension of the objects that undergo transformations under the group's action.

To achieve this, it becomes essential to identify the Cartan subalgebra, which encompasses the maximum count of independent operators that can be simultaneously diagonalized. In this context, we introduce the Casimir operator, which plays a crucial role in the study of the group's representation:

$$\mathbf{J}^2 = J^i J^i. \tag{2.16}$$

This operator commutes with all the generators:

$$\left[\mathbf{J}^2, J_a\right] = 0. \tag{2.17}$$

According to a corollary of Schur's lemma, $\mathbf{J}^2$ must be proportional to the identity operator. The proportionality constant, denoted as C_2, may vary depending on the specific representation. Thus, we have:

$$\mathbf{J}^2 = C_2 \mathbb{1}_n, \tag{2.18}$$

where the constant C_2 is referred to as the quadratic Casimir. Given that $\left[\mathbf{J}^2, J_a\right] = 0$, the elements of the Cartan subalgebra in our case are $\left\{\mathbf{J}^2, J^3\right\}$.

Now, we build a basis of states to set up the eigenvalue problem. As in quantum mechanics, we label the states by $|j, m\rangle$, where m is the eigenvalue of J^3:

$$J^3 |j, m\rangle = m |j, m\rangle, \tag{2.19}$$

and j is a common label for all the states that belong to the same multiplet, which coincides with the largest eigenvalue of J^3, such that

$$J^3 |j, j\rangle = j |j, j\rangle. \tag{2.20}$$

Now, we define the ladder operators:

$$J^{\pm} \equiv J^1 \pm i J^2. \tag{2.21}$$

In terms of these operators, the $\mathfrak{su}(2)$ algebra becomes:

$$\left[J^3, J^{\pm}\right] = \pm J^{\pm}, \quad \left[J^+, J^-\right] = 2J^3, \tag{2.22}$$

and due to hermiticity:

$$(J^-)^\dagger = J^+. \tag{2.23}$$

We compute the action of $J^3 J^\pm$ on the states $|j,m\rangle$:

$$J^3 J^\pm |j,m\rangle = \left(J^\pm J^3 \pm J^\pm\right)|j,m\rangle = (m \pm 1) J^\pm |j,m\rangle, \tag{2.24}$$

meaning that the state $J^\pm|j,m\rangle$ is an eigenvector of J^3 with eigenvalue $m \pm 1$. Therefore, $J^\pm|j,m\rangle$ must be proportional to the state $|j,m\pm 1\rangle$. Since the maximum value of m is j, we must have:

$$J^+|j,j\rangle = 0. \tag{2.25}$$

It can be shown that the following relation holds:

$$\langle j,m|\left[\mathbf{J}^2 - (J^3)^2\right]|j,m\rangle = C_2 - m \geq 0. \tag{2.26}$$

This relation implies the existence of a lower bound for the lowest value of m. Consequently, denoting this state as j', there is a state annihilated by J^-:

$$J^-|j,j'\rangle = 0. \tag{2.27}$$

It can be shown that the corresponding values of C_2 and j' are:

$$C_2 = j(j+1), \quad j' = -j. \tag{2.28}$$

Finally, both ends of the ladder must be connected by a finite number of steps. Thus, for some integer n, we have:

$$\left(J_+\right)^n |j,-j\rangle \sim |j,j\rangle \Rightarrow -j + n = j \Rightarrow j = \frac{n}{2}. \tag{2.29}$$

This means that j takes semi-integer values.
For the states to be normalized, we must have:

$$J^+|j,m\rangle = \sqrt{(j-m)(j+m+1)}|j,m+1\rangle, \tag{2.30}$$

$$J^-|j,m\rangle = \sqrt{(j+m)(j-m+1)}|j,m-1\rangle. \tag{2.31}$$

Summarizing, the irreducible representation of SU(2) is determined by the relations:

$$\begin{aligned} \mathbf{J}^2|j,m\rangle &= j(j+1)|j,m\rangle, \quad J^3|j,m\rangle = m|j,m\rangle, \\ J^{\pm}|j,m\rangle &= \sqrt{(j\mp m)(j\pm m+1)}|j,m\pm 1\rangle, \\ j = \frac{n}{2}, &\quad n\in\mathbb{Z}, \quad m = -j,-j+1,\dots,j-1,j. \end{aligned} \tag{2.32}$$

The explicit form of an irreducible representation of SU(2) is determined by either the value of j or the dimensionality of the representation. The rotation matrix is given by:

$$D_{(j)}(\theta,\hat{\mathbf{n}}) = e^{-i\mathbf{J}_{(j)}\cdot\hat{\mathbf{n}}\theta}, \tag{2.33}$$

which is a $(2j+1)\times(2j+1)$ matrix for the irreducible representation of spin j. Additionally, the associated generators are also $(2j+1)\times(2j+1)$ matrices, whose elements are given by:

$$\begin{aligned} \left\langle j,m'|J^3_{(j)}|j,m\right\rangle &= m\delta_{m',m}, \\ \left\langle j,m'|J^+_{(j)}|j,m\right\rangle &= \sqrt{(j+m+1)(j-m)}\delta_{m',m+1}, \\ \left\langle j,m'|J^-_{(j)}|j,m\right\rangle &= \sqrt{(j+m)(j-m+1)}\delta_{m',m-1}. \end{aligned} \tag{2.34}$$

Let's work out the generators for the simplest irreducible representations.
The minimal value that j can take is $j=0$. This corresponds to the trivial representation, which is a representation of dimension one:

$$|0,0\rangle = 1, \tag{2.35}$$

$$J^i_{(0)} = 0 \quad \rightarrow \quad D_{(0)}(\theta,\hat{\mathbf{n}}) = 1. \tag{2.36}$$

For $j=1/2$, we have the fundamental representation:

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad \left|\frac{1}{2},-\frac{1}{2}\right\rangle = \begin{pmatrix}0\\1\end{pmatrix}, \tag{2.37}$$

$$J^3_{(1/2)} = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad J^+_{(1/2)} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \quad J^-_{(1/2)} = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}. \tag{2.38}$$

In terms of the Cartesian components, the generators for $j=1/2$ are given by:

$$J^1 = \frac{1}{2}\left(J^- + J^+\right) = \frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{2.39}$$

$$J^2 = \frac{1}{2}\left(J^- - J^+\right) = \frac{1}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \tag{2.40}$$

and we recover

$$J^i_{(1/2)} = \frac{\sigma^i}{2}. \tag{2.41}$$

For $j = 1$, we have the adjoint representation:

$$|1,1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad |1,0\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad |1,-1\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}, \tag{2.42}$$

$$J^1_{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}, \quad J^2_{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix}, \quad J^3_{(1)} = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}. \tag{2.43}$$

2.1.3 Direct product of irreducible representations

We consider two independent SU(2) systems, denoted as A and B, each with its own algebra:

$$\begin{aligned}\left[J^i_A, J^j_A\right] &= i\epsilon_{ijk}J^k_A,\\ \left[J^i_B, J^j_B\right] &= i\epsilon_{ijk}J^k_B,\\ \left[J^i_A, J^j_B\right] &= 0.\end{aligned} \tag{2.44}$$

The system undergoes transformations under the direct product group $\mathrm{SU}(2)_A \otimes \mathrm{SU}(2)_B$. Our chosen set of complete commuting observables consists of $\{\mathbf{J}^2_A, J^3_A, \mathbf{J}^2_B, J^3_B\}$. Consequently, the states can be expressed as:

$$\left|j^A, m^A\right\rangle \otimes \left|j^B, m^B\right\rangle \equiv \left|j^A, m^A; j^B, m^B\right\rangle. \tag{2.45}$$

The $\mathrm{SU}(2)_A \otimes \mathrm{SU}(2)_B$ representations can be derived by taking the direct product of individual SU(2) irreducible representations:

$$D(\theta, \hat{\mathbf{n}}) = D_{(j^A)}(\theta, \hat{\mathbf{n}}) \otimes D_{(j^B)}(\theta, \hat{\mathbf{n}}). \tag{2.46}$$

These representations are generally reducible. Therefore, it is advantageous to describe the system using an alternative set of observables, which incorporates the total angular momentum:

$$\mathbf{J} = \mathbf{J}_A \otimes \mathbb{I}_B + \mathbb{I}_A \otimes \mathbf{J}_B. \tag{2.47}$$

An alternative complete commuting set of observables is $\{\mathbf{J}^2, J^3, \mathbf{J}_A^2, \mathbf{J}_B^2\}$. We label the states as $|J, M, j^A, J^B\rangle \equiv |JM\rangle$. This basis satisfies

$$\mathbf{J}^2|JM\rangle = J(J+1)|JM\rangle, \quad J^3|JM\rangle = M|JM\rangle, \tag{2.48}$$

and facilitates the decomposition of the direct product representations into irreducible ones

$$D(\theta, \hat{\mathbf{n}}) = D_{(j^A)}(\theta, \hat{\mathbf{n}}) \otimes D_{(j^B)}(\theta, \hat{\mathbf{n}}) = \bigoplus_{J=|j^B-j^A|}^{j^A+j^B} D_{(J)}(\theta, \hat{\mathbf{n}}). \tag{2.49}$$

Now, it is essential to establish the connection between the eigenvalues and their ranges and determine the Clebsch-Gordan coefficients that underpin the linear relationship between both bases. It can be shown that

$$J^3|j^A, m^A\rangle|j^B, m^B\rangle = (m^A + m^B)|j^A, m^A\rangle|j^B, m^B\rangle. \tag{2.50}$$

Furthermore, the ladder operators $J_\pm$ establish connections between all states that transform under the same irreducible representation, denoted by J. The maximum value of J is determined by j^A+j^B, while the minimum value is given by $J = |j^B-j^A|$. For each value of J, the parameter M assumes values between $-J$ and J in increments of size one.

2.1.4 The algebra of the SU(3) group

The SU(3) group is characterized as the set of 3×3 complex unitary matrices U with a determinant equal to 1. The algebra's dimension is $\dim \mathrm{SU}(3) = 8$. One frequently employed defining representation is expressed using the Gell-Mann matrices λ_a, analogous to the role played by the Pauli matrices for the algebra of the generators of the SU(2) group:

$$\begin{aligned}
&\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
&\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
&\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{aligned} \tag{2.51}$$

The generators X_a of the su(3) Lie algebra are then

$$X_a = \frac{1}{2}\lambda_a. \tag{2.52}$$

These generators are normalized as follows:

$$\mathrm{tr}(X_a X_b) = \frac{1}{2}\delta_{ab}. \tag{2.53}$$

The $\mathfrak{su}(3)$ algebra reads

$$[X_a, X_b] = i f_{abc} X_c, \tag{2.54}$$

where the fully antisymmetric structure constants with nonzero entries are

$$\begin{gathered} f_{123} = 1, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}, \\ f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2}. \end{gathered} \tag{2.55}$$

2.2 The Standard model of particle physics

The Standard Model (SM) is a quantum field theory that stands on a robust phenomenological foundation. It exhibits a remarkable predictive power in terms of various particle properties and interactions[31, 32]. Within the SM framework, three out of the four fundamental forces of nature are explained as arising from interactions via gauge bosons.

2.2.1 Standard Model ingredients

In the Standard Model, we employ fields to represent elementary particles, with each elementary particle being a quantum of the corresponding quantum field. Based on their spins, elementary particles can be classified into elementary bosons and elementary fermions. Most elementary bosons are spin-1 gauge vector bosons, and they function as the force carriers for the fundamental interactions.

Fermionic particles, on the other hand, interact with each other by exchanging these gauge vector bosons. This exchange of gauge bosons mediates the interactions between fermions, leading to the rich array of phenomena observed in particle physics[33]. The foundation of the Standard Model lies in the local symmetry group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, where the indices have no mathematical meaning, but they refer to color, weak isospin, and hypercharge, respectively.

This gauge structure uniquely governs the dynamics of the strong, weak, and electromagnetic interactions that take place between matter particles and force carriers. The coupling constants associated with these interactions are precisely determined through experimental investigations, thus providing a robust framework that successfully accounts for a wide range of particle physics phenomena.

The key ingredients of the Standard Model are:

- **Vector fields:** 8 non-abelian gauge bosons associated with $\mathrm{SU}(3)_c$ known as gluons, represented by G_a^μ, with $a = 1, \dots, 8$; 3 non-abelian gauge bosons associated with $\mathrm{SU}(2)_J$ denoted by W_i^μ, with $i = 1, 2, 3$; and an abelian one corresponding to $\mathrm{U}(1)_Y$ denoted by B^μ. Gluons mediate the strong interactions, the weak interactions are mediated by the charged W and Z bosons, and photons mediate electromagnetic interactions.

- **Spin 1/2 fermion fields:** Left-handed Weyl spinors transforming as isodoublets (Left-handed Weyl spinors belong in the representation $(1/2, 0)$ of the Lorentz algebra):

$$\begin{aligned} L_{iL} &= \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \right\}_L, \\ Q_{iL} &= \left\{ \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \right\}_L, \end{aligned} \tag{2.56}$$

  and right-handed Weyl spinors transforming as isosinglets:

$$e_{iR} = \{e, \mu, \tau\}_R, \quad u_{iR} = \{u, c, t\}_R, \quad d_{iR} = \{d, s, b\}_R. \tag{2.57}$$

  There are two kinds of fermion fields: quarks, transforming as triplets under $\mathrm{SU}(3)_C$, and leptons, which are singlets under color.

- **Scalar fields:** One isodoublet transforming as a singlet under color:

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}. \tag{2.58}$$

The SM fields and their transformation properties are summarized in the following table:

Field	SU(3)	SU(2)	U(1)
Q_{iL}	**3**	**2**	1/6
u_{iR}	**3**	**1**	2/3
d_{iR}	**3**	**1**	−1/3
L_{iL}	**1**	**2**	−1/2
e_{iR}	**1**	**1**	-1
H	**1**	**2**	1/2

Table 2.1: SM transformation properties

The representation of the fields is usually denoted as (a, b, c), meaning that the given field is an a-plet under $\mathrm{SU}(3)_c$, b-plet under $\mathrm{SU}(2)_L$, and has hypercharge c.

2.2.2 Glashow-Weinberg-Salam Electroweak Model

The Standard Model is also known as the Glashow-Weinberg-Salam model, and our main focus will be the electroweak part of the theory. We can express the compact form of the Standard Model Lagrangian as follows:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}, \tag{2.59}$$

which includes the gauge, fermion, Higgs, and Yukawa sectors of the theory. In our present focus on the electroweak portion of the model, described by $\mathrm{SU}(2) \times \mathrm{U}(1)$, we designate the generators of $\mathrm{SU}(2)$ as T_i, and the generator of $\mathrm{U}(1)$ as Y. The algebra of the generators satisfies the following relations:

$$\left[T_i, T_j\right] = i\epsilon_{ijk} T_k, \quad [T_i, Y] = 0. \tag{2.60}$$

For the gauge bosons, we define the matrix value fields

$$\mathcal{W}^\mu = T_i W_i^\mu. \tag{2.61}$$

The electric charge generator is embedded into the electroweak symmetry according to the Gell-Mann-Nishijima formula

$$Q = T_3 + Y. \tag{2.62}$$

The local invariance of the SM Lagrangian is implemented by the covariant derivative

$$D^\mu = \partial^\mu + ig\mathcal{W}^\mu + ig' Y B^\mu = \partial^\mu + igT_i W_i^\mu + ig' Y B^\mu. \tag{2.63}$$

where g and g' are the $\mathrm{SU}(2)$ and $\mathrm{U}(1)$ couplings, respectively.

The kinetic Lagrangian density for fermion fields is

$$\mathcal{L}_{\text{fermion}} = \bar{L}_L i\gamma^\mu D_\mu L_L + \bar{e}_R i\gamma^\mu D_\mu e_R + \bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R. \tag{2.64}$$

Explicitly, for leptons, we have

$$\mathcal{L}_{\text{lepton}} = \bar{L}_L i\gamma^\mu \left(\partial_\mu + \frac{i}{2} g\tau_i W_{\mu i} - \frac{i}{2} g' B_\mu\right) L_L + \bar{e}_R i\gamma^\mu \left(\partial_\mu - ig' B_\mu\right) e_R \tag{2.65}$$

and for quarks

$$
\begin{aligned}
\mathcal{L}_{\text{quark}} =& \bar{Q}_L i\gamma^\mu \left(\partial_\mu + \frac{i}{2} g \tau^a A^a_\mu + \frac{i}{6} g' B_\mu\right) Q_L \\
&+ \bar{u}_R i\gamma^\mu \left(\partial_\mu + \frac{2i}{3} g' B_\mu\right) u_R + \bar{d}_R i\gamma^\mu \left(\partial_\mu - \frac{i}{3} g' B_\mu\right) d_R.
\end{aligned} \tag{2.66}
$$

At this point, all particles are considered massless. The inclusion of mass terms would break gauge invariance. To include mass terms for fermions, the Higgs mechanism[34] for Spontaneous Symmetry Breaking is necessary and will be introduced in the next section.

The kinetic lagrangian density for gauge fields is

$$
\mathcal{L}_{\text{gauge}} = -\frac{1}{2} \operatorname{Tr}\left[\mathcal{W}^{\mu\nu}\mathcal{W}_{\mu\nu}\right] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{2.67}
$$

where

$$
\mathcal{W}^{\mu\nu} = D^\mu \mathcal{W}^\nu - D^\nu \mathcal{W}^\mu, \tag{2.68}
$$

$$
B^{\mu\nu} = D^\mu B^\nu - D^\nu B^\mu. \tag{2.69}
$$

We can also express the Lagrangian term for the charged current interaction, which involves the non-diagonal generators of SU(2), as follows:

$$
\mathcal{L}_{\text{charged}} = -\frac{g}{2} \left\{ \bar{L}_L \gamma^\mu \left(\tau_1 W_{\mu 1} + \tau_2 W_{\mu 2}\right) L_L + \bar{Q}_L \gamma^\mu \left(\tau_1 W_{\mu 1} + \tau_2 W_{\mu 2}\right) Q_L \right\}. \tag{2.70}
$$

We define

$$
W^\pm_\mu = \frac{1}{\sqrt{2}} \left(W_{\mu 1} \mp i W_{\mu 2}\right) \tag{2.71}
$$

to obtain

$$
W_{\mu 1} = \frac{1}{\sqrt{2}} \left(W^+_\mu + W^-_\mu\right), \quad W_{\mu 2} = \frac{i}{\sqrt{2}} \left(W^+_\mu - W^-_\mu\right), \tag{2.72}
$$

which enables us to write

$$
\begin{aligned}
\mathcal{L}_{\text{charged}} =& -\frac{g}{\sqrt{2}} \left\{ \bar{L}_L \gamma^\mu \left[\frac{\tau_1}{2}\left(W^+_\mu + W^-_\mu\right) + \frac{i\tau_2}{2}\left(W^+_\mu - W^-_\mu\right)\right] L_L \right. \\
&\left. + \bar{Q}_L \gamma^\mu \left[\frac{\tau_1}{2}\left(W^+_\mu + W^-_\mu\right) + \frac{i\tau_2}{2}\left(W^+_\mu - W^-_\mu\right)\right] Q_L \right\}.
\end{aligned} \tag{2.73}
$$

It can be demonstrated that by defining the charged currents as:

$$\begin{aligned} J_W^{\mu} &= 2\left[\bar{e}_L\gamma^{\mu}\nu_L + \bar{d}_L\gamma^{\mu}u_L\right], \\ J_W^{\dagger\mu} &= 2\left[\bar{\nu}_L\gamma^{\mu}e_L + \bar{u}_L\gamma^{\mu}d_L\right], \end{aligned} \tag{2.74}$$

the above expression simplifies to:

$$\mathcal{L}_{\text{charged}} = -\frac{g}{2\sqrt{2}}\left[J_W^{\mu}W_{\mu}^{-} + J_W^{\dagger\mu}W_{\mu}^{+}\right]. \tag{2.75}$$

The Lagrangian term for neutral current interaction is given by:

$$\begin{aligned} \mathcal{L}_{\text{neutral}} =& - g\left[\bar{L}_L\gamma^{\mu}\frac{\tau_3}{2}W_{\mu 3}L_L + \bar{Q}_L\gamma^{\mu_3}\frac{\tau_3}{2}W_{\mu 3}Q_L\right] \\ &+ \frac{g'}{2}\left[\bar{L}_L\gamma^{\mu}B_{\mu}L_L + 2\bar{e}_R\gamma^{\mu}B_{\mu}e_R\right] \\ &- \frac{g'}{2}\left[\frac{1}{3}\bar{Q}_L\gamma^{\mu}B_{\mu}Q_L + \frac{4}{3}\bar{u}_R\gamma^{\mu}B_{\mu}u_R - \frac{2}{3}\bar{d}_R\gamma^{\mu}B_{\mu}d_R\right]. \end{aligned} \tag{2.76}$$

In component fields, the leptonic sector can be rewritten as:

$$\begin{aligned} \mathcal{L}_{\text{leptons}}^{\text{neutral}} =& \bar{\nu}_L\gamma^{\mu}\nu_L\left(-\frac{g}{2}W_{\mu 3} + \frac{g'}{2}B_{\mu}\right) \\ &+ \bar{e}_L\gamma^{\mu}e_L\left(\frac{g}{2}W_{\mu 3} + \frac{g'}{2}B_{\mu}\right) + g'\bar{e}_R\gamma^{\mu}e_R B_{\mu}, \end{aligned} \tag{2.77}$$

while for quarks, we have

$$\begin{aligned} \mathcal{L}_{\text{quarks}}^{\text{neutral}} =& \bar{u}_L\gamma^{\mu}u_L\left(-\frac{g}{2}W_{\mu 3} - \frac{g'}{6}B_{\mu}\right) + \bar{d}_L\gamma^{\mu}d_L\left(\frac{g}{2}W_{\mu 3} - \frac{g'}{6}B_{\mu}\right) \\ &- g'\left[\frac{2}{3}\bar{u}_R\gamma^{\mu}u_R B_{\mu} - \frac{1}{3}\bar{d}_R\gamma^{\mu}d_R B_{\mu}\right]. \end{aligned} \tag{2.78}$$

The Higgs potential is included in the $\mathcal{L}_{\text{Higgs}}$ term and triggers the spontaneous symmetry breaking. The term $\mathcal{L}_{\text{Yukawa}}$ contains the Yukawa interaction between the Higgs and SM fermions. These processes are responsible for mass generation and will be studied in the following sections.

2.2.3 Spontaneous Symmetry Breaking

As mentioned earlier, fermions are initially considered massless in the Standard Model. However, directly introducing mass terms would violate gauge invariance, a fundamental symmetry of the model. To incorporate mass terms for fermions while preserving gauge invariance, the Higgs mechanism for Spontaneous Symmetry Breaking (SSB) is employed.

To understand how it works, we study the scalar sector, described by the Lagrangian density:

$$\mathcal{L}_{\text{Higgs}} = (D^\mu H)^\dagger \left(D_\mu H\right) - V(H), \tag{2.79}$$

where H is the Higgs doublet introduced in (2.58). The spontaneous symmetry breaking of the electroweak theory occurs when the potential is chosen as:

$$V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2, \tag{2.80}$$

with $\mu^2 > 0$ and $\lambda > 0$. There are infinitely many degenerate vacua characterized by a continuous phase α, and the vacuum expectation value (VEV) is given by:

$$\langle H \rangle = \frac{v e^{i\alpha}}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{2.81}$$

Without loss of generality, we can choose $\alpha = 0$, resulting in:

$$\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{2.82}$$

The value of v is determined by the minimum condition:

$$0 = \left.\frac{\partial V}{\partial\left(\sqrt{2}\,\mathrm{Re}\, h^0\right)}\right|_{H=\langle H \rangle} = \frac{\partial}{\partial v}\left(-\frac{1}{2}\mu^2 v^2 + \frac{1}{4}\lambda v^4\right) = -\mu^2 v + \lambda v^3. \tag{2.83}$$

In the broken phase, the only solution is $v^2 = \frac{\mu^2}{\lambda}$. This non-zero VEV is what gives us the so-called spontaneous symmetry breaking.
For the VEV to be compatible with the preservation of $\mathrm{U}(1)_{\mathrm{Q}}$ after Electroweak Symmetry Breaking (EWSB), it must be invariant under an infinitesimal U(1) transformation:

$$e^{iQ\epsilon}\langle H \rangle = (1 + iQ\epsilon)\langle H \rangle = \langle H \rangle \tag{2.84}$$

or, equivalently, Q must annihilate the vacuum $Q\langle H\rangle = 0$. Explicitly,

$$\begin{aligned} Q\langle H\rangle &= (T_3+Y)\langle H\rangle \\ &= \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0. \end{aligned} \tag{2.85}$$

As observed, the non-zero VEV of the Higgs field retains the conservation of electric charge, maintaining the unaltered charge symmetry. However, this VEV breaks the electroweak gauge symmetry in the following manner:

$$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \to \mathrm{U}(1)_\mathrm{Q}, \tag{2.86}$$

A convenient parametrization for the scalar doublet is

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} = e^{-iT_i'\xi_i(x)/v}\begin{pmatrix} 0 \\ \frac{\rho}{\sqrt{2}} \end{pmatrix}, \tag{2.87}$$

where the T_i' are the three broken generators T_1, T_2 and $T_3 - Y$, and the fields ξ_i, ρ are real.
To identify the particle spectrum of the Standard Model, it is advantageous to work in the unitary gauge, involving unitary gauge transformations $U_{\mathrm{SU}(2)} = e^{i\tau_i\theta_i(x)/2}$ and $U_{\mathrm{U}(1)} = e^{-\theta/2}$, where $\theta_i(x) = \xi_i(x)/v$ and $\theta = \xi_3(x)/v$. Under these gauge transformations, the Higgs field H transforms as follows:

$$H \to U_{\mathrm{SU}(2)}U_{\mathrm{U}(1)}H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}, \tag{2.88}$$

where h represents the physical Higgs boson. In this choice of gauge, only the physical degrees of freedom appear in the Lagrangian. The fields ξ_i are the Nambu-Goldstone bosons associated with the breakdown of 3 out of 4 generators of the electroweak symmetry that are absorbed by the gauge fields and ultimately become the longitudinal degrees of freedom of the massive physical vector fields.
It can be shown that in the unitary gauge, the kinetic Lagrangian for H takes the form

$$\mathcal{L}_H^{\mathrm{kin}} = \frac{1}{2}\partial^\mu h\partial_\mu h + \frac{1}{8}\left[2g^2W^{\mu+}W_\mu^- + (-gW_3^\mu + g'B^\mu)^2\right](v+h)^2. \tag{2.89}$$

Simplifying, we obtain

$$\begin{aligned}\mathcal{L}_H^{\text{kin}} =&\frac{1}{2}\partial^\mu h\partial_\mu h + \frac{g^2}{4}(v+h)^2 W^{\mu+}W_\mu^- \\ &+ \frac{(g^2+g'^2)}{8}(v+h)^2\left(\frac{gW_3^\mu - g'B^\mu}{\sqrt{g^2+g'^2}}\right)^2 .\end{aligned} \tag{2.90}$$

The last term can be further simplified by noticing that:

$$\left(\frac{gW_3^\mu - g'B^\mu}{\sqrt{g^2+g'^2}}\right)^2 = \frac{1}{g^2+g'^2}\begin{pmatrix} W_3^\mu & B^\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_{\mu 3} \\ B_\mu \end{pmatrix}, \tag{2.91}$$

which can be diagonalized by an orthogonal transformation of the form:

$$\begin{pmatrix} W_{\mu 3} \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{2.92}$$

where $c_W \equiv \cos\theta_W$, $s_W \equiv \sin\theta_W$, and θ_W is the weak (or Weinberg) angle. The weak angle can be determined explicitly as follows:

$$\begin{aligned}&\left(\frac{gW_3^\mu - g'B^\mu}{\sqrt{g^2+g'^2}}\right)^2 \\ &= \frac{1}{g^2+g'^2}\begin{pmatrix} Z^\mu & A^\mu \end{pmatrix}\begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{1}{g^2+g'^2}\begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \\ &\begin{pmatrix} (gc_W + g's_W)^2 & -gg'\cos 2\theta_W + \frac{(g^2-g'^2)}{2}\sin 2\theta_W \\ -gg'\cos 2\theta_W + \frac{(g^2-g'^2)}{2}\sin 2\theta_W & (gs_W - g'c_W)^2 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}.\end{aligned} \tag{2.93}$$

From the diagonal condition, we obtain:

$$\tan 2\theta_W = \frac{2gg'}{g^2 - g'^2}. \tag{2.94}$$

By identifying A^μ with the massless photon, we find:

$$\tan\theta_W \equiv t_W = \frac{g'}{g}. \tag{2.95}$$

The solution to both equations is

$$s_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{2.96}$$

In terms of the physical gauge bosons, the kinetic scalar term becomes

$$\mathcal{L}_H^{\text{kin}} = \frac{1}{2}\partial^\mu h \partial_\mu h + \frac{g^2}{4}(v+h)^2 W^{\mu+} W_\mu^- + \frac{(g^2 + g'^2)}{8}(v+h)^2 Z^\mu Z_\mu. \tag{2.97}$$

We can read the mass terms for the mediators of the weak interaction

$$\frac{g^2 v^2}{4} W^{\mu+} W_\mu^- + \frac{1}{2}\frac{(g^2 + g'^2)\, v^2}{4} Z^\mu Z_\mu = M_W^2 W^{\mu+} W_\mu^- + \frac{1}{2} M_Z^2 Z^\mu Z_\mu, \tag{2.98}$$

where:

$$M_W^2 = \frac{g^2 v^2}{4}, \quad M_Z^2 = \frac{(g^2 + g'^2)\, v^2}{4}, \tag{2.99}$$

and these masses are related by $M_W = c_W M_Z$.

The $W^\pm$ and Z bosons mediate the weak interactions. The $W^\pm$ bosons, discovered in 1983[35], are the charged partners of the weak force, while the Z boson, also discovered in 1983[36], is the neutral counterpart. Notably, these massive vector bosons are directly linked to broken symmetries. The strong force carriers (gluons) and photons remain massless, while the weak force carriers ($W^\pm$ and Z bosons) acquire mass due to spontaneous symmetry breaking through the Higgs mechanism.

Expressing the Higgs potential in terms of the VEV, we have:

$$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 - \frac{\lambda v^4}{4}, \tag{2.100}$$

which, in the unitary gauge, becomes:

$$V(H) = \lambda \left(v^2 h^2 + v h^3 + \frac{h^4}{4} \right) - \frac{\lambda v^4}{4}. \tag{2.101}$$

Analyzing the quadratic term in h, we can determine the squared mass of the physical scalar:

$$\frac{1}{2} M_H^2 h^2 = \lambda v^2 h^2 \quad \Rightarrow M_H^2 = 2\lambda v^2. \tag{2.102}$$

This mass corresponds to the Higgs boson observed in 2012[37]. The measured value is $M_H = 125.25 \pm 0.17$ GeV[33].

2.2.4 Fermion masses

To unravel the mechanism behind mass generation for Standard Model fermions, a careful examination of the Yukawa interaction term is essential. These interactions encapsulate all possible contractions between scalars and fermions while preserving the defining symmetries of the model. The Yukawa interactions are embodied in the Lagrangian term:

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} =& -\bar{L}_L H y_e e_R - \bar{e}_R y_e^\dagger H^\dagger L_L - \bar{Q}_L H y_d d_R - \bar{d}_R y_d^\dagger H^\dagger Q_L \\ & -\bar{Q}_L \tilde{H} y_u u_R - \bar{u}_R y_u^\dagger \tilde{H}^\dagger Q_L.\end{aligned} \tag{2.103}$$

Here, the matrices y_e, y_u, and y_d are 3×3 constant complex matrices. Additionally, $\tilde{H}$ denotes the charge conjugate of H, defined as:

$$\tilde{H} = i\tau_2 H^* = \begin{pmatrix} h^0 \\ -h^- \end{pmatrix}, \quad h^- = h^{+*}, \tag{2.104}$$

where $Y(\tilde{H}) = -1/2$ represents the hypercharge of the charge conjugate scalar field $\tilde{H}$.

After EWSB, mass terms are induced for all charged fermions in the model. As usual, the mass spectrum is easily found in unitary gauge, where

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} &= -\bar{L}_L H y_e e_R - \bar{Q}_L H y_d d_R - \bar{Q}_L \tilde{H} y_u u_R + \text{ h.c.} \\ &= -\frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \begin{pmatrix} 0 \\ v+h \end{pmatrix} y_e e_R - \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \begin{pmatrix} 0 \\ v+h \end{pmatrix} y_d d_R \\ &- \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \begin{pmatrix} v+h \\ 0 \end{pmatrix} y_u u_R + \text{ h.c.} \\ &= -\bar{e}_L \frac{y_e(v+h)}{\sqrt{2}} e_R - \bar{d}_L \frac{y_d(v+h)}{\sqrt{2}} d_R - \bar{u}_L \frac{y_u(v+h)}{\sqrt{2}} u_R + \text{ h.c.}\end{aligned} \tag{2.105}$$

In general, the mass matrices for the charged fermions in the Standard Model are given by:

$$M_e = y_e \frac{v}{\sqrt{2}}, \quad M_d = y_d \frac{v}{\sqrt{2}}, \quad M_u = y_u \frac{v}{\sqrt{2}}. \tag{2.106}$$

These mass matrices are generally non-diagonal 3×3 complex matrices, implying that the interaction states and the states with well-defined masses (mass eigenstates) do not necessarily coincide. This means that the fermion fields that participate in the weak interactions, known as the interaction states, are not the same as the fields that have definite masses, known as the mass eigenstates. In the case of neutrinos, which were considered to be massless for a long time, it is possible to redefine the fields in a way that makes the mass matrix M_e diagonal without any physical implications.

Transitioning to quark Yukawa interactions, let's consider the Standard Model fields in the interaction basis, denoted by a superscript 0. Using this notation, the quark Yukawa interactions in the unitary gauge can be expressed as:

$$\mathcal{L}_{\text{Yuk}}^{\text{quark}} = -\bar{u}_L^0 M_u \left(1 + \frac{h}{v}\right) u_R^0 - \bar{d}_L^0 M_d \left(1 + \frac{h}{v}\right) d_R^0 + \text{h.c.} \tag{2.107}$$

To identify the physical fields, we need to diagonalize the complex matrices M_u and M_d through a bi-unitary transformation:

$$M^u = V_L^u m^u V_R^{u\dagger}, \quad M^d = V_L^d m^d V_R^{d\dagger}, \tag{2.108}$$

where $V_{L,R}^u$ and $V_{L,R}^d$ are 3×3 unitary matrices, and m^u, m^d are diagonal matrices with non-negative entries. This bi-unitary transformation defines a basis for the mass eigenstate fields as:

$$u_{L,R} \equiv {V_{L,R}^u}^\dagger u_{L,R}^0, \quad d_{L,R} \equiv {V_{L,R}^d}^\dagger d_{L,R}^0. \tag{2.109}$$

In this new basis, the quark Yukawa interactions simplify to:

$$\mathcal{L}_{\text{Yuk}}^{\text{quark}} = -\bar{u}_L m_u \left(1 + \frac{h}{v}\right) u_R - \bar{d}_L m_d \left(1 + \frac{h}{v}\right) d_R + \text{h.c.} \tag{2.110}$$

We can now express the quark gauge interactions in terms of the mass eigenstates. For the neutral currents, the Lagrangian is given by:

$$\mathcal{L}_{\text{quarks}}^{\text{neutral}} = \left[-eJ_Q^\mu A\mu - \frac{g}{2c_W} J_Z^\mu Z_\mu\right]_{\text{quarks}}, \tag{2.111}$$

where $J_Q^\mu\Big|_{\text{quarks}} = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d$ represents the electromagnetic current and $J_Z^\mu|_{\text{quarks}} = \bar{u}_L\gamma^\mu u_L - \bar{d}L\gamma^\mu d_L - 2s_W^2 J_Q^\mu\Big|_{\text{quarks}}$ represents the weak neutral current.

This implies that both the electromagnetic current and the weak neutral current have the same form in the mass and flavor basis. As a result, both currents are flavor diagonal and family universal. This feature, known as the GIM (Glashow-Iliopoulos-Maiani) mechanism[38], was originally introduced to suppress unobserved flavor-changing effects and predict the existence of the charm quark. Without the GIM mechanism, the d and s quarks would not have the same electroweak quantum numbers, leading to flavor-changing-neutral currents.

In contrast, for the charged currents, the Lagrangian is given by:

$$
\begin{aligned}
\mathcal{L}^{\text{charged}}_{\text{quarks}} &= -\frac{g}{\sqrt{2}}\left[\bar{d}^0_L\gamma^\mu u^0_L W^-_\mu + \bar{u}^0_L\gamma^\mu d^0_L W^+_\mu\right] \\
&= -\frac{g}{\sqrt{2}}\left[\bar{d}_L\gamma^\mu V^{d\dagger}_L V^u_L u_L W^-_\mu + \bar{u}_L\gamma^\mu V^{u\dagger}_L V^d_L d_L W^+_\mu\right] \\
&\equiv -\frac{g}{\sqrt{2}}\left[\bar{d}_L\gamma^\mu V^\dagger_{\text{CKM}} u_L W^-_\mu + \bar{u}_L\gamma^\mu V_{\text{CKM}} d_L W^+_\mu\right],
\end{aligned} \tag{2.112}
$$

where we have introduced the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix[39, 40]:

$$
V_{\text{CKM}} = V^{u\dagger}_L V^d_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2.113}
$$

which describes the mismatch between the unitary transformations relating the weak and mass eigenstates for the up and down quarks. By convention, we absorb the action of V_{CKM} on d-type quarks, which allows us to write:

$$
\mathcal{L}^{\text{charged}}_{\text{quarks}} = -\frac{g}{\sqrt{2}}\left[\left(\bar{u}_L\gamma^\mu d^{\text{mix}}_L + \bar{c}_L\gamma^\mu s^{\text{mix}}_L + \bar{t}_L\gamma^\mu b^{\text{mix}}_L\right) W^+_\mu + \text{h.c.}\right], \tag{2.114}
$$

where we define the mixed states:

$$
\begin{pmatrix} d \\ s \\ b \end{pmatrix}^{\text{mix}}_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L . \tag{2.115}
$$

The CKM matrix, V_{CKM}, can be parametrized using three mixing angles and one CP violating complex phase, which encode the flavor-changing weak decays of quarks and play a crucial role in flavor physics phenomenology. The most popular parametrization of the CKM matrix, as provided by the Particle Data Group (PDG)[33], is given by:

$$
\begin{aligned}
V_{\text{CKM}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{+i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&\quad c_{12}c_{13} \\
&= \begin{pmatrix} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix},
\end{aligned} \tag{2.116}
$$

where $s_{ij} \equiv \sin\theta_{ij}$, and $c_{ij} \equiv \cos\theta_{ij}$ represent the mixing angles, and δ_{13} is a CP violating phase.

2.2.5 Radiative Corrections and Anomaly Cancellation

Feynman diagrams devoid of loops represent tree-level processes. However, at higher orders, corrections known as radiative corrections come into play. In such cases, the corresponding Feynman diagrams involve loops. Under certain circumstances, these radiative corrections have the potential to disrupt the symmetries inherent in classical equations of motion. A classical symmetry affected by quantum effects is deemed anomalous.

Noether's theorem establishes that continuous global symmetries imply the existence of conserved currents. However, when a symmetry becomes anomalous, it ceases to be a genuine symmetry, and the associated current is no longer conserved. In the realm of unitary quantum theories, gauged symmetries must be anomaly-free. This requirement holds true in the SM, where the electric charge is compelled to be quantized, and the charges of quarks and leptons are interrelated.

Anomalies linked to gauge bosons are referred to as gauge anomalies. In cases where a symmetry is not gauged, anomalies pose no issues. Global anomalies, such as those associated with baryon and lepton number conservation, do not result in inconsistencies within the SM.

For chiral fermions, the cancellation of chiral anomalies is imperative to ensure the consistency of the theory. Symmetries under which left and right-handed fields transform identically are termed vector symmetries, while those transforming with opposite charges are referred to as chiral symmetries. The currents associated with these symmetries are denoted as $J^\mu = \overline{\psi}\gamma^\mu\psi$ and $J^{\mu 5} = \overline{\psi}\gamma^\mu\gamma^5\psi$, known as the vector current and axial current, respectively.

At the one-loop level, in the presence of a gauge coupling to fermions, $J^{\mu 5}$ is not conserved, leading to an anomalous breaking of chiral symmetry[41]. For non-abelian gauge theories, the currents associated with gauge fields take the form $J^a_\mu = \sum_\psi \overline{\psi}_i T^a_{ij}\gamma^\mu\psi_j$, where T^a_{ij} represents the group generators.

From the summation of two triangle diagrams resulting in the chiral anomaly, terms of the form

$$\operatorname{tr}\left[T^a T^b T^c\right] = \frac{1}{2}\operatorname{tr}\left[\left[T^a, T^b\right] T^c\right] + \frac{1}{2}\operatorname{tr}\left[\left\{T^a, T^b\right\} T^c\right] = i\frac{1}{2}T_R f^{abc} + \frac{1}{4}d^{abc}_R \tag{2.117}$$

can be derived. It can be shown that

$$\operatorname{tr}\left[T^a_R\left\{T^b_R, T^c_R\right\}\right] = A(R)\operatorname{tr}\left[T^a\left\{T^b, T^c\right\}\right] \equiv A(R)d^{abc}, \tag{2.118}$$

where $A(R)$ is the anomaly coefficient. The divergence of the current is given by

$$\partial_\alpha J^a_\alpha(x) = \left(\sum_{\text{left}} A(R_l) - \sum_{\text{right}} A(R_r)\right) \frac{g^2}{128\pi^2} d^{abc} \varepsilon^{\mu\nu\alpha\beta} F^b_{\mu\nu} F^c_{\alpha\beta} \tag{2.119}$$

Upon considering each fermion within the theory, the diverse contributions must mutually cancel for every conceivable combination of generators from each gauge symmetry, encompassing the mixing of different gauge symmetries. In the SM, any anomaly with precisely one factor of SU(2) or SU(3) is effectively canceled.

In the context of combinations involving two $\mathrm{SU}(3)_c$ bosons and one $\mathrm{U}(1)_Y$ boson, the anomaly arises solely from quarks. Utilizing the property $\operatorname{tr}\{T^a T^b\} = \frac{1}{2}\delta^{ab}$, we can determine that

$$\operatorname{tr}\left[T^a \left\{T^b, Y\right\}\right] = 6Y_Q - 3Y_u - 3Y_d = 6 \cdot \frac{1}{6} - 3 \cdot \frac{2}{3} - 3 \cdot \frac{-1}{3} = 0. \tag{2.120}$$

For the $\mathrm{SU}(2)^2\,\mathrm{U}(1)$ anomaly, contributions only emerge from left-handed fields, resulting in

$$\operatorname{tr}\left[\tau^a \left\{\tau^b, Y\right\}\right] = 2Y_L + 6Y_Q = 2 \cdot \frac{-1}{2} + 6 \cdot \frac{1}{6} = 0. \tag{2.121}$$

Addressing the $\mathrm{U}(1)^3$ anomaly, the expression becomes:

$$(2Y_L^3 - Y_e^3 - Y_\nu^3) + 3\left(2Y_Q^3 - Y_u^3 - Y_d^3\right) = 2\left(-\frac{1}{2}\right)^3 - (-1)^3 + 3(2)(\frac{1}{6})^3 - 3\left(\frac{2}{3}\right)^3 - 3\left(-\frac{1}{3}\right)^3 = 0. \tag{2.122}$$

Ensuring the cancellation of graviton anomalies in the SM focuses on $\mathrm{grav}^2\,\mathrm{U}(1)_Y$:

$$(2Y_L - Y_e - Y_\nu) + 3\left(2Y_Q - Y_u - Y_d\right) = 2\left(-\frac{1}{2}\right) - (-1) + 3\left(2(\frac{1}{6}) - \left(\frac{2}{3}\right) - \left(-\frac{1}{3}\right)\right) = 0. \tag{2.123}$$

Thus, every anomalous contribution vanishes, ensuring the consistency of the SM.

It is noteworthy that baryon number, where quarks have $B = 1$ and leptons $B = 0$, and lepton number, where quarks have $L = 0$ and leptons $L = 1$, are anomalous in the SM. However, the global symmetry $B - L$, with quarks having $B - L = \frac{1}{3}$ and leptons $B - L = -1$, remains non-anomalous.

2.2.6 The Lorentz Group

This section presupposes a basic understanding of special relativity, omitting certain definitions and focusing on aspects pertinent to subsequent chapters. The Lorentz group constitutes the most comprehensive set of transformations that preserves the Minkowski metric: $\Lambda^T g \Lambda = g$.

Within this group, fundamental transformations include rotations and boosts. Any Lorentz group element can be uniquely expressed as:

$$\Lambda = \exp\left(-i\theta_i J_i - i\beta_i K_i\right), \tag{2.124}$$

where J_i represents the generators of rotations, and K_i denotes the generators of boosts.

Two discrete Lorentz transformations, known as parity and time reversal, are noteworthy. The parity transformation, denoted as P, entails changing the sign of spatial coordinates while leaving the time coordinate unchanged:

$$P : (t, x, y, z) \to (t, -x, -y, -z). \tag{2.125}$$

Conversely, the time reversal transformation, denoted as T, involves changing the sign of the time coordinate while keeping spatial coordinates constant:

$$T : (t, x, y, z) \to (-t, x, y, z). \tag{2.126}$$

The collective group of translations and Lorentz transformations is referred to as the Poincaré group, denoted as $\mathrm{ISO}(1,3)$. The Lorentz group itself is sometimes denoted as $\mathrm{O}(1,3)$.

In relativistic quantum field theory, particles are described by irreducible unitary representations of the Poincaré group. The Lie algebra of a group is defined by the commutation relations among its generators. For the Lorentz group, the generators of rotations and boosts satisfy the following commutation relations:

$$\begin{aligned} \left[J_i, J_j\right] &= i\epsilon_{ijk} J_k, \\ \left[J_i, K_j\right] &= i\epsilon_{ijk} K_k, \\ \left[K_i, K_j\right] &= -i\epsilon_{ijk} J_k. \end{aligned} \tag{2.127}$$

The generators of the Lorentz group constitute the algebra corresponding to the part connected to the identity, referred to as the proper orthochronous Lorentz group ($\mathrm{O}^+(1,3)$). The proper Lorentz group $\mathrm{SO}(1,3)$ encompasses elements with a determinant equal to 1.

Scalar fields, denoted as $\phi(x)$, are functions of spacetime that remain invariant under Lorentz transformations, preserving their form. The transformation of a scalar field under a Lorentz transformation Λ^μ_ν is expressed as:

$$\phi(x) \to \phi((\Lambda^{-1})^\mu_\nu x^\nu). \tag{2.128}$$

In contrast, 4-vectors V^μ transform under Lorentz transformations as:

$$V^\mu \to \Lambda^\mu_\nu V^\nu. \tag{2.129}$$

The generators A_i and B_i are derived as linear combinations of the generators of rotations (J_i) and boosts (K_i):

$$A_i \equiv \frac{1}{2}(J_i + iK_i), \quad B_i \equiv \frac{1}{2}(J_i - iK_i). \tag{2.130}$$

These generators satisfy the commutation relations:

$$\begin{aligned}\left[A_i, A_j\right] &= i\epsilon_{ijk}A_k,\\ \left[B_i, B_j\right] &= i\epsilon_{ijk}B_k,\\ \left[A_i, B_j\right] &= 0.\end{aligned} \tag{2.131}$$

These commutation relations illustrate that the Lie algebra for the Lorentz group can be decomposed into two commuting subalgebras:

$$\mathrm{so}(1,3) = \mathrm{su}(2) \oplus \mathrm{su}(2). \tag{2.132}$$

Each irreducible representation of su(2) is characterized by a half-integer value j, acting on a vector space with $2j + 1$ basis elements. Accordingly, irreducible representations of the Lorentz group are defined by two half-integer values denoted as a and b, where the (a, b) representation has $(2a + 1)(2b + 1)$ degrees of freedom.

For example, there exist two complex representations of $J = \frac{1}{2}$ denoted as $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{2}\right)$. The Pauli matrices, σ_i, satisfy the commutation relations:

$$\left[\sigma_i, \sigma_j\right] = 2i\epsilon_{ijk}\sigma_k. \tag{2.133}$$

Rescaling the Pauli matrices, we get:

$$\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\epsilon_{ijk}\frac{\sigma_k}{2}. \tag{2.134}$$

The generators of the Lorentz algebra so$(1,3)$ in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations can be expressed as:

$$\left(\frac{1}{2}, 0\right): \quad \mathbf{A} = \frac{1}{2}\boldsymbol{\sigma}, \quad \mathbf{B} = \mathbf{0}, \tag{2.135}$$

$$\left(0, \frac{1}{2}\right): \quad \mathbf{A} = \mathbf{0}, \quad \mathbf{B} = \frac{1}{2}\boldsymbol{\sigma}. \tag{2.136}$$

It is also true that the generators of rotations are given by $\mathbf{J} = \mathbf{A} + \mathbf{B}$, and the generators of boosts are $\mathbf{K} = i(\mathbf{B} - \mathbf{A})$. Consequently, we find:

$$\begin{aligned} \left(\frac{1}{2}, 0\right): \quad & \mathbf{J} = \frac{1}{2}\boldsymbol{\sigma}, \quad \mathbf{K} = -\frac{i}{2}\boldsymbol{\sigma}, \\ \left(0, \frac{1}{2}\right): \quad & \mathbf{J} = \frac{1}{2}\boldsymbol{\sigma}, \quad \mathbf{K} = \frac{i}{2}\boldsymbol{\sigma}. \end{aligned} \tag{2.137}$$

The elements of the vector space on which the spin-$\frac{1}{2}$ representations act are referred to as spinors. In this context, the $\left(\frac{1}{2}, 0\right)$ spinors are known as left-handed Weyl spinors, while the $\left(0, \frac{1}{2}\right)$ spinors are termed right-handed Weyl spinors. Fields are spinor-valued functions of spacetime, represented as:

$$\psi_R(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}, \tag{2.138}$$

for the $(0, \frac{1}{2})$ representation. Similarly, we denote $\psi_L(x)$ for the $(\frac{1}{2}, 0)$ representation. The $\left(\frac{1}{2}, 0\right)$ representation acts on ψ_L as

$$\psi_L \to e^{-\frac{1}{2}(i\boldsymbol{\theta}\cdot\boldsymbol{\sigma} + \boldsymbol{\beta}\cdot\sigma)}\psi_L, \tag{2.139}$$

and similarly for ψ_R:

$$\psi_R \to e^{-\frac{1}{2}(i\boldsymbol{\theta}\cdot\boldsymbol{\sigma} - \boldsymbol{\beta}\cdot\sigma)}\psi_R. \tag{2.140}$$

These two spinors can be combined into a four-component object known as a Dirac spinor:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \tag{2.141}$$

Armed with the concepts and tools introduced so far, the next chapter will explore mass generation models for neutrinos, which is an intriguing topic in particle physics. Various mechanisms have been proposed to elucidate the tiny but non-zero neutrino masses observed in experiments. But first, a quick review of dark matter is presented.

2.3 Dark Matter

This section draws heavily from the insights presented in Majumdar's book[42]. A remarkable aspect of our universe is the prevalence of dark matter (DM), constituting approximately 26% of its total content. The term “dark matter” originates from the fact that this substance behaves like matter from the point of view of the Friedmann equations, in the sense that it does not apply pressure, but it is not charged under electromagnetic interaction, and therefore it is not observable. The model to be presented in this thesis introduces candidates for dark matter, prompting a concise exploration of this mysterious entity. Given the considerable efforts invested in developing theories to unravel the nature of dark matter, it is crucial to address why there exists a high level of certainty regarding its existence. Numerous pieces of evidence support the existence of dark matter. In the subsequent sections, we will briefly explore key proofs that contribute to the compelling argument supporting the existence of dark matter.

2.3.1 Evidence of dark matter

An apparent incongruity emerges when comparing the observable mass and the gravitational mass in the universe, hinting at the existence of a substantial unseen mass. In galaxies, stars exhibit a relatively uniform circular motion around their center. This circular motion's velocity seemingly precisely counterbalances the gravitational force directed towards the galactic center, maintaining the stars in their circular orbits. For a star positioned at distance r from the galactic center, with a circular velocity $v(r)$, the equilibrium of gravitational and centrifugal forces is expressed as

$$\frac{mv(r)^2}{r} = \frac{Gm}{r^2}\int_0^r \rho(r')dr', \tag{2.142}$$

where $\rho(r)$ denotes the galaxy's mass density. The rotation curve, indicating the radial dependence of stellar orbital velocities, reveals an unexpected observation. While the mass density is anticipated to decrease with distance, empirical measurements of rotation curves for various galaxies show a constant velocity for large r, suggesting the presence of a substantial unseen mass forming a surrounding halo.

Another compelling piece of evidence for dark matter arises from the study of galaxy clusters, gravitational bound groups of galaxies. The inference of dark matter within these clusters involves estimating their mass based on gravitational dynamics, juxtaposed with mass estimates derived from luminosity. Typically, the virial theorem is employed to gauge the gravitating mass, revealing a discrepancy when compared to the luminous mass. A historical instance involves Zwicky's 1933 computation[43] of the mass-to-luminosity ratio for the Coma cluster and individual galaxies, exposing a cluster ratio about 50 times greater than that of any single galaxy.

Einstein's general relativity describes the concept of gravitational lensing, where light bends in the presence of gravitating mass. Astronomical observations of this phenomenon, without concurrent detection of luminous mass, point to the existence of dark matter. The scrutiny of the cluster 1E0657-56, colloquially known as the bullet cluster, stands as a noteworthy testament to dark matter. In this case, the utilization of weak and strong lensing observations helped delineate the distribution of dark matter.

In addition to astronomical evidence, significant cosmological insights into DM arise from pivotal moments in the early universe's evolution. Observations related to Big Bang Nucleosynthesis (BBN), the Cosmic Microwave Background (CMB)[44], energy density scaling of matter, radiation, the cosmological constant, and supernovae type-Ia observations[45] collectively highlight that ordinary baryonic matter alone cannot account for the entirety of the universe's mass. Moreover, the inclusion of DM in the cosmic energy budget proves essential for comprehending structure formation and evolution.

2.3.2 Dark matter candidates

While the precise nature of dark matter remains elusive, categorization is possible based on its potential production mechanisms, the particle characteristics of its constituents, or the mass of DM candidates. Essential criteria for a viable DM candidate, as outlined by Taoso et al.[46], include neutrality under electromagnetic and strong interactions, compliance with self-interaction constraints, stability over timescales comparable to the universe's age, and consistency with the appropriate relic density.

Dark matter can be classified based on whether it underwent thermal or non-thermal production in the early universe. Thermal production involves DM generation through cosmic plasma collisions during the radiation-dominated era. In contrast, non-thermal dark matter particles may originate from alternative mechanisms, such as the decay of massive particles or specific symmetry conditions.

DM candidates can be categorized based on their mass and velocity characteristics. When DM exhibits relativistic speeds, it falls into the category of hot dark matter, possessing a mass less than the Universe's temperature at a relevant time. Conversely, if the DM mass exceeds the temperature of the universe at freeze-out, it is termed cold dark matter (CDM).

Within the Standard Model, at least two of the known three neutrino species are non-relativistic and contribute to the universe's matter content, qualifying them as potential DM candidates. However, their transition to a non-relativistic state occurred at very late times, impacting the radiation budget crucial for structure formation. As a result, they are considered hot DM. Observations of structures discount the possibility that neutrinos constitute the entire DM abundance, despite contributing a small fraction. This limitation prompts the search for DM candidates beyond the SM. Popular alternatives include axions, sterile neutrinos, and Weakly Interacting Massive Particles (WIMPs).

A noteworthy example of non-thermal DM is the axion, introduced as a resolution to the strong-CP problem. The Peccei-Quinn mechanism not only addressed the strong-CP problem but also predicted a new massive particle—the QCD axion. While this specific axion variant has been excluded by particle collider experiments, alternative QCD axion models remain viable candidates for dark matter.

Sterile neutrinos represent another potential dark matter (DM) candidate. These particles lack any charge under SM interactions and, in the simplest scenario, only interact with SM particles through weak interactions. This interaction is suppressed by their mixing angle with active neutrinos, and sterile neutrinos are exempt from self-interactions. Typically, sterile neutrinos are anticipated to have a mass on the order of keV and could generate x-rays through radiative decays of active neutrinos. However, cosmological and x-ray observations have imposed constraints on the allowed parameter space for sterile neutrinos to comprise the DM.

Weakly Interacting Massive Particles (WIMPs) offer another avenue for DM candidates. These generic massive particles, besides gravitational interactions, exhibit only weak coupling to the Standard Model[47]. Initially, WIMPs were postulated with an interaction strength comparable to the weak interaction in the SM[48]. This assumption, termed "The WIMP Miracle", relied on the coincidence that a massive particle with thermal relic abundance and an interaction cross-section on the scale of the SM weak interaction would naturally yield the correct relic abundance order of magnitude. The term "weakly interacting" is now applied more broadly, encompassing particles that exhibit weak coupling to SM particles, lack direct photon coupling, and were thermally produced in the early universe, with their relic density determined by their freeze-out abundance. Given the diversity of particle DM candidates within the WIMP category, a more comprehensive discussion on this candidate follows.

2.3.3 WIMP Dark Matter and Relic Density

If dark matter candidates were in thermal and chemical equilibrium in the early Universe, they underwent decoupling from the universal plasma when the interaction rates became less than the expansion rate of the Universe, leading to a constant comoving density for such particles. In this context, Weakly Interacting Massive Particles (WIMPs) emerge as interesting dark matter candidates. The annihilation cross-section, deduced from experimental assessments of dark matter abundance (relic density), aligns with the expected range for weak interaction cross-sections.

WIMPs, once in chemical and thermal equilibrium at sufficiently high temperatures in the early Universe, were thermally produced through collisions in the thermal cosmic plasma. This occurred when they were generated in particle-antiparticle pairs. Subsequently, these particle-antiparticle pairs could annihilate, forming Standard Model particles in a reverse reaction. Initially, these two processes were in equilibrium, denoting the dark matter particle as χ and its number density as n_χ, resulting in $n_\chi - \overline{n}_\chi = 0$.

Expressing the number density of such particles in terms of the Boltzmann distribution function for temperatures T and mass m_χ:

$$n_\chi - \bar{n}_\chi \sim \left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-m_\chi/T}. \tag{2.143}$$

As the annihilation rate drops just below the expansion rate, the number of these particles in a comoving volume stabilizes. This phase, termed “freeze-out”, results in the particles lingering as relics, with the temperature at which this occurs known as the “freeze-out temperature” for that particle species. The relic density subsequent to freeze-out is dependent on the annihilation cross-section.

In the calculation of relic densities for thermally produced DM candidates, the Boltzmann equation needs to be solved:

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} = -3Hn_\chi - \langle\sigma v\rangle \left(n_\chi^2 - \left(n_\chi\right)_{eq}^2\right), \tag{2.144}$$

where H denotes the Hubble constant, and $\langle\sigma v\rangle$ represents the thermally averaged product of the annihilation cross-section and the relative velocity between the DM particles. This parameter serves as a bridge between Cosmology and Particle Physics, dictating the DM density post freeze-out. Once the Boltzmann equation is solved, the estimated DM density can be expressed as[49]:

$$\Omega_\chi h^2 \approx \frac{3\times 10^{-27}\,[\ \mathrm{cm}^3\ \mathrm{s}^{-1}]}{\langle\sigma v\rangle}. \tag{2.145}$$

The measured value for this quantity is $\Omega_\chi h^2 = 0.11425 \pm 0.00311$[50]. This equation provides a means to test the WIMP hypothesis for a given DM particle candidate.

Current observations strongly indicate that dark matter is cold. Consequently, relativistic particles cannot account for the DM abundance within the present cosmological framework. Even if we disregard the relativistic nature of neutrinos, the neutrino relic density remains significantly lower than the DM relic density. Hence, the abundance of dark matter in the universe cannot be justified by the Standard Model.

3

Massive neutrinos

Contents

The recognition of neutrinos dates back to the 1956 Cowan-Reines neutrino experiment[51]. As stated in the last chapter, neutrinos remain massless in the SM even after the spontaneous symmetry breaking. In the present day, we identify three distinct types of neutrinos, all electrically neutral and exclusively interacting through the weak force. This chapter delves into the intriguing realm of neutrino masses. Despite being predicted to be at least six orders of magnitude smaller than the next lightest standard model fermion, experimental evidence has unequivocally demonstrated that neutrinos possess tiny but non-zero masses.

The first section of this chapter elucidates the theoretical underpinnings of neutrinos, shedding light on the experiments that have substantiated their minute yet non-negligible masses. We explore various mechanisms proposed to account for the origin of neutrino masses, recognizing that their massiveness necessitates physics beyond the Standard Model (BSM).

In the subsequent section, we introduce an extension to the Standard Model that not only accommodates neutrino masses but also posits a candidate for dark matter.

3.1 Introduction to neutrino masses

We begin this section with some basic theory about fermions to introduce the concept of chirality, needed to approach the discussion of whether neutrinos are Dirac or Majorana particles. We continue this section with the concept of neutrino oscillation, which requires neutrinos to be massive and that their masses are non-degenerate. Afterwards, the most popular mechanisms for neutrino mass generation are presented.

3.1.1 Chirality and fermions

The Dirac equation is a relativistic wave equation that describes the behavior of fermionic particles with spin-1/2, such as electrons. It is given by:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0, \tag{3.1}$$

where the γ^μ are the 4×4 gamma matrices, namely a Dirac representation of the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{3.2}$$

In the Weyl (or chiral) representation, the gamma matrices take the following form:

$$\gamma^0 = \begin{pmatrix} & \mathbb{1} \\ \mathbb{1} & \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}. \tag{3.3}$$

The Dirac matrices can also be expressed in an alternative form as:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}. \tag{3.4}$$

The Dirac equation is Lorentz invariant, and if ψ satisfies the Dirac equation, it automatically satisfies the Klein-Gordon equation, which is a relativistic wave equation describing spin-0 particles:

$$\begin{aligned}(i\gamma^\mu\partial_\mu + m)(i\gamma^\nu\partial_\nu - m)\psi &= \left(-\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu + m^2 - im(\gamma^\mu\partial^\mu + \gamma^\nu\partial_\nu) - m^2\right)\psi \\ &= \left(-\frac{1}{2}\left\{\gamma^\mu,\gamma^\nu\right\}\partial_\mu\partial_\nu - \frac{1}{2}\left[\gamma^\mu,\gamma^\nu\right]\partial_\mu\partial_\nu - m^2\right)\psi \\ &= -\left(\partial_\mu\partial^\mu + m^2\right)\psi = 0.\end{aligned} \tag{3.5}$$

The Dirac adjoint, denoted as $\overline{\psi}$, is a useful construct in combining spinors to form Lorentz invariant quantities. It is defined as the Hermitian conjugate of the Dirac spinor ψ multiplied by the gamma matrix γ^0:

$$\overline{\psi} \equiv \psi^\dagger\gamma^0. \tag{3.6}$$

Using the Dirac adjoint, we can construct various Lorentz invariant quantities, such as $\overline{\psi}\gamma^\mu\psi$, $\overline{\psi}\psi^\mu\psi^\nu\psi$, and $\overline{\psi}\partial_\mu\psi$. These expressions remain invariant under Lorentz transformations due to the proper combination of Dirac spinors and gamma matrices. The Dirac Lagrangian, given by

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi, \tag{3.7}$$

leads directly to the Dirac equation when the Euler-Lagrange equation is applied. The general solution to the Dirac equation is a complex 4-vector transforming under the spin representation of the Lorentz group. However, this solution is not irreducible.
The gamma matrix γ^5 is defined as:

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma, \tag{3.8}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor with $\epsilon^{0123} = -\epsilon_{0123}$. In the chiral representation, γ^5 is diagonal:

$$\gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}, \tag{3.9}$$

and it satisfies several important properties:

- Hermiticity: $\left(\gamma^5\right)^\dagger = \gamma^5$,

- Idempotency: $(\gamma^5)^2 = \mathbb{1}$,

- Anti-commutation with gamma matrices: $\{\gamma^5, \gamma^\mu\} = 0$.

The matrix γ^5 is known as the chiral or helicity projection operator because it projects the Dirac spinor into its left-handed and right-handed components. To show this, we introduce the projectors

$$P_{L,R} = \frac{1}{2}\left(\mathbb{1} \mp \gamma^5\right). \tag{3.10}$$

They satisfy $P_{L,R}^2 = P_{L,R}$ and $P_L P_R = 0$. When acting with these projectors on the Dirac spinor ψ, we obtain its left-handed and right-handed components:

$$\psi_L = P_L\psi, \quad \psi_R = P_R\psi, \tag{3.11}$$

with $\psi = \psi_L + \psi_R$. These left-handed and right-handed components are known as Weyl spinors, often referred to as the chiral components. Using the chiral components, we can express the Lagrangian the form:

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - \overline{\psi}m\psi = i\overline{\psi}_L\gamma^\mu\partial_\mu\psi_L + \overline{\psi}_R\gamma^\mu\partial_\mu\psi_R - \overline{\psi}_L m\psi_R - \overline{\psi}_R m\psi_L. \tag{3.12}$$

From the Lagrangian, we can derive the field equations for the chiral components:

$$\begin{aligned} i\gamma^\mu\partial\psi_R &= m\psi_L, \\ i\gamma^\mu\partial\psi_L &= m\psi_R. \end{aligned} \tag{3.13}$$

If we set $m = 0$, the space-time evolutions of the Weyl spinors ψ_R and ψ_L decouple, yielding the Weyl equations:

$$\begin{aligned} i\gamma^\mu\partial\psi_R &= 0, \\ i\gamma^\mu\partial\psi_L &= 0. \end{aligned} \tag{3.14}$$

Since the field equations for ψ_R and ψ_L are now decoupled, it is possible that one of the two chiral fields is sufficient to describe a massless fermion.

All known matter particles in the SM are known to be Dirac fermions, except for neutrinos, which could be Majorana particles. To understand the difference between Dirac and Majorana fermions, we need to introduce the charge conjugation operator, denoted by C, which satisfies the relations[52]:

$$C^\dagger = C^T = C^{-1} = -C, \quad C\gamma^\mu C = -(\gamma^\mu)^T. \tag{3.15}$$

The charge-conjugated field ψ^c is defined as $\psi^c = C\psi^\dagger$. We note that the action of C on a Weyl fermion flips its chirality: $(\psi_L)^c = (\psi^c)_R$. This means that the antiparticle of a left-handed particle is right-handed and vice-versa.
We now define the Majorana spinor, which satisfies the Majorana condition:

$$\psi^c \equiv C\overline{\psi}^T = \psi. \tag{3.16}$$

Due to the action of C that inverts all charge-like quantum numbers, Majorana fermions must carry zero charge. It can be shown that a Majorana fermion can be written as the sum of a Weyl fermion and its complex conjugate. Consequently, the Majorana mass term takes the following form:

$$-\overline{\psi}_L m\psi_R - \overline{\psi}_R m\psi_L = -\overline{\psi}_L m\overline{\psi}^c_L + \text{h.c.} \tag{3.17}$$

Here, h.c. denotes the Hermitian conjugate. Furthermore, it is possible to rewrite the Dirac mass terms in the following manner:

$$\overline{\Psi_R} m\Psi_L + \overline{\Psi_L} m\Psi_R = \frac{1}{2}\left[(\Psi^c)^T_L Cm\Psi_L + \Psi^T_L Cm^T (\Psi^c)_L\right] + \text{ h.c.} \tag{3.18}$$

This rewriting reveals that a Dirac fermion can be viewed as a combination of two Majorana fermions with the same mass but opposite CP parity, a phenomenon known as maximal mixing. However, introducing Majorana masses for neutrinos within the framework of the Standard Model (SM) is not straightforward. An $\text{SU}(2)_L$ transformation of ψ_L would not preserve the Lagrangian mass term. Accomplishing this requires the introduction of something akin to the Higgs mechanism or an effective operator. Unfortunately, none of these options can be implemented within the SM without violating gauge invariance, Lorentz invariance, or renormalizability. Consequently, it is not possible to consistently introduce neutrino mass terms within the rules of the SM.

3.1.2 Neutrino oscillations

While the Standard Model initially portrayed neutrinos as massless particles, the notion that massive neutrinos could exhibit flavor-changing properties was postulated by Pontecorvo in 1957[53, 54]. At that time, only one type of neutrino was known, but the subsequent discovery of a second type of neutrino[55] fueled the development of theories describing neutrino mixing between different flavor eigenstates[56, 57].

The experimental landscape took a significant turn with the advent of neutrino oscillation experiments, providing compelling evidence for non-zero neutrino masses. The first inkling of this came in 1968 with the detection of solar neutrinos[58]. The subsequent discovery of the tau neutrino in 2000[59] and the pivotal data from Super-Kamiokande[5] and SNO[15] conclusively established that neutrinos can change their flavor during propagation.

Neutrinos are primarily produced through charged-current weak interactions, yielding weak-eigenstate neutrinos (ν_e, ν_μ, or ν_τ). However, the neutrino mass matrix is not generally diagonal in this basis. Consequently, the mass eigenstate neutrinos (ν_1, ν_2, and ν_3) may differ from the flavor eigenstates, setting the stage for flavor oscillations over time. therefore, the probability of finding a neutrino created in a certain flavour state to be in the same or other state will oscillate with time.

We consider first the case of Dirac neutrino mass term. In this case, the part of the Lagrangian thar describes lepton masses and charged current interactions can be written as

$$-\mathcal{L}_{W+m} = \frac{g}{\sqrt{2}}\bar{e}_L\gamma^\mu\nu_L W_\mu^- + m_l\bar{e}_L e_R + m_D\bar{\nu}_L\nu_R + \text{ h.c.}, \tag{3.19}$$

where we have used the flavour eigenstate fields. We note that the individual lepton flavours are not conserved when the Dirac neutrino mass term is present, but the total lepton number L is. In general, the mass matrices m_l and m_D are complex, anc can be diagonalized via bi-unitary transformations. We write

$$e_L = V_l e'_L, \quad e_R = V_R e'_R, \quad \nu_L = U_L\nu'_L, \quad \nu_R = U_R\nu_R, \tag{3.20}$$

where the matrices V_L, V_R, U_L and U_R are chosen so that they diagonalize the mass matrices of the charged leptons and neutrinos. To keep the notation simple, we will omit the primes when changing to the Dirac mass eigenstates $e_i = e_{iL} + e_{iR}$ and $\nu_i = \nu_{iL} + \nu_{iR}$. The Lagrangian then takes the form

$$-\mathcal{L}_{W+m} = \frac{g}{\sqrt{2}}\bar{e}\gamma^\mu V_L^\dagger U_L\nu_L W_\mu^- + m_l\bar{e}_L e_R + m_D\bar{\nu}_L\nu_R + \text{ h.c.}, \tag{3.21}$$

where now m_l and m_D represent the charged lepton masses and neutrino masses, respectively. The matrix that relates neutrino flavour states and mass states is called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix: $U = V_L^\dagger U_L$. The relationship between neutrino flavour eigenstates $|\nu_i\rangle$ produced or absorbed alongside with the corresponding charged lepton, to the mass eigenstates ν_k has the form:

$$|\nu_i\rangle = \sum_k U_{ik}^*|\nu_k\rangle, \tag{3.22}$$

where $i \in e, \mu, \tau$ denotes the neutrino flavour, ν_k are the neutrino mass eigenstates, and the U_{ik}

is is an element of the unitary PMNS matrix. The matrix U satisfies the unitary conditions:

$$\left(UU^{\dagger}\right)_{\alpha\beta} = \sum_i U_{\alpha i} U^*_{\beta i} = \delta_{\alpha\beta}, \quad \left(U^{\dagger}U\right)_{ij} = \sum_\alpha U^*_{\alpha i} U_{\alpha j} = \delta_{ij}. \tag{3.23}$$

The PMNS matrix, in the Particle Data Group (PDG) convention, is given by:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \mathrm{diag}\left(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}\right), \tag{3.24}$$

where c_{ij} and s_{ij} are the cosine and sine of the mixing angle between the i-th flavour eigenstate and j-th mass eigenstates, respectively. The symbol δ represents the Dirac CP-violating phase, and α_{21} and α_{31} are the Majorana CP-violating phases. Let us consider an initial state produced at $t = 0$ by some charged current (CC) process involving a neutrino with flavor state α. The evolution operator describes the state at a time t as follows:

$$|\nu_\alpha(t)\rangle = \sum_k U^*_{\alpha k} e^{-iE_k t} |\nu_k\rangle = \sum_k U^*_{\alpha k} e^{-iE_k t} \sum_\alpha U_{\beta k} |\nu_\beta\rangle. \tag{3.25}$$

The probability of the neutrino flavor transition $\nu_\alpha \to \nu_\beta$ is given by:

$$P\big(\nu_\alpha \to \nu_\beta, t\big) = \left|\big\langle \nu_\beta \mid \nu_\alpha(t)\big\rangle\right|^2 = \left|\sum_k U^*_{\alpha k} U_{\beta k} e^{-iE_k t}\right|^2 = \sum_{kj} \left(J_{\alpha\beta}\right)_{kj} e^{-i(E_k - E_j)t}, \tag{3.26}$$

where we have defined the self-adjoint matrix $\left(J_{\alpha\beta}\right)_{kj} = U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}$. The factor $U^*_{\alpha k}$ is interpreted as the amplitude of transformation of the initial flavour eigenstate neutrino ν_α into a mass eigenstate one ν_k. The factor $U_{\beta k}$ converts the time-evolved mass eigenstate ν_k into the flavour eigenstate ν_β. In the ultra-relativistic limit, we can approximate the energy eigenvalues E_k as

$$E_k \approx E + \frac{m_k^2}{2E}. \tag{3.27}$$

Inserting this approximation into equation (3.26) yields:

$$P\big(\nu_\alpha \to \nu_\beta, t\big) = \sum_{kj} \left(J_{\alpha\beta}\right)_{kj} \exp\left(i\frac{\Delta m^2_{jk} L}{2E}\right), \tag{3.28}$$

where $L = ct$ is the distance between the neutrino source and the detector, and $\Delta m^2_{jk} = m_j^2 - m_k^2$ is the squared mass difference between the neutrino mass eigenstates.

The case for Majorana neutrinos is analogous, the difference is that the term $m_D \overline{\nu}_L \nu_R + \text{h.c.}$

needs to be replaced by $m_M \overline{\nu}_L^c \nu_R + \text{h.c.}$ But in this case, not only the individual lepton flavour is broken, but also the total lepton number. The structure of the charged current interactions turns out to be the same as in the case of Dirac neutrinos. Therefore, the oscillation probabilities in the case of the Majorana mass term are the same as in the case of the Dirac mass term. This means that we cannot distinguish between Dirac and Majorana neutrinos by observing neutrino oscillations. The situation changes when the neutrino mass term is of the Dirac+Majorana form. In this case, again, total lepton number conservation is violated by the Majorana mass term. Unlike the previous cases, we can have a new type of neutrino oscillation: oscillation between sterile states $\nu_a \to \nu_b^c$ can also occur. In principle, this case can be distinguished from the pure Dirac and Majorana cases in neutrino oscillations experiments[60].

In order to observe neutrino oscillations, it is necessary to have a non-degenerate mass spectrum, which implies $\Delta m_{jk}^2 \neq 0$, and a non-trivial flavor mixing matrix, denoted by $U \neq \mathbb{1}$. Today there is a strong body of evidence of neutrino oscillations from a variety of experiments. However, neutrino oscillation experiments are sensitive to the difference between squared neutrino masses only, and thus are insensitive to the absolute neutrino mass scale. Today, the square mass difference Δm_{21}^2 is known and is positive, yielding $m_2 > m_1$, but nothing has been discovered yet about the sign of Δm_{32}^2. This leads to two mass schemes, named the normal ordering (NO): $m_1 < m_2 < m_3$, and the inverted ordering (IO): $m_3 < m_1 < m_2$.

The absolute neutrino mass scale can be probed by neutrinoless double-beta decay ($0\nu\beta\beta$)[61] laboratory experiments. $0\nu\beta\beta$ decay is a process where two neutrons beta-decay simultaneously without emitting any neutrinos or antineutrinos. This process violates lepton number by two units. Therefore, neutrinos must be Majorana in order to induce $0\nu\beta\beta$ decay[62]. If the neutrinos are Dirac, then this process will be absent and the Majorana phases in the PMNS matrix are non-physical and can be set to zero. In addition, observation of this process would also provide invaluable information about the dominance of matter over antimatter in the Universe, because two matter particles (electrons) are emitted in the decay without the balance of the corresponding antiparticles[63].

We discuss briefly the consequences of CP, T and CPT symmetries for neutrino oscillations. Note that charge conjugation operation C is not well defined for neutrinos, as it would convert a left handed neutrino into a non-existent left handed antineutrino. On the contrary, CP is well defined: it converts a left handed neutrino ν_L into a right handed antineutrino. CP is essentially particle-antiparticle conjugation. If CP is conserved, the probabilities of oscillations between particles and their antiparticles coincide:

$$P(\nu_\alpha \to \nu_\beta, t) = P(\overline{\nu}_\alpha \to \overline{\nu}_\beta, t)\,. \tag{3.29}$$

The action of particle-antiparticle conjugation on the matrix U amounts to $U \to U^*$, which means that CP is only conserved in the leptonic sector if the matrix U is real or can be made real by a rephasing of the lepton fields. A unitary $n \times n$ matrix depends on $n(n-1)/2$ angles and $n(n+1)/2$ phases. In the Dirac case, $2n-1$ phases can be removed by proper rephasing of the left fields, which leaves $(n-1)(n-2)/2$ physical phases. Therefore, in the Dirac case, CP non-conservation is only possible for $n \geq$ generations. In the Majorana case, only n phases can be removed, leaving $n(n-1)/2$ physical phases. The Majorana mass phases do not lead to observable effects for neutrino oscillations[64].

CPT transformation can be considered as a combined action of CP and time reversal, which interchanges the initial and final states. Under CPT, the oscillation probability $P\big(\nu_\alpha \to \nu_\beta, t\big)$ goes into $P\big(\overline{\nu}_\beta \to \overline{\nu}_\alpha, t\big)$. But under CPT, $U \to U^*$ and $t \to -t$, which transforms the oscillation amplitude into its complex conjugate. Thus, oscillation probabilities under CPT are invariant with respect to CPT:

$$P\big(\nu_\alpha \to \nu_\beta, t\big) = P\big(\overline{\nu}_\beta \to \overline{\nu}_\alpha, t\big)\,. \tag{3.30}$$

From CPT invariance, it follows that CP conservation is equivalent to T conservation. If CP is not conserved, oscillation probabilities are different for neutrinos from those for antineutrinos. This is possible if the matrix U is complex, i.e. it has unremovable phases. For three generations there is only one such phase δ, so there should be only one CP-odd oscillation asymmetry. Denoting the CP-odd asymmetry as

$$\Delta P_{\alpha\beta} \equiv P\big(\nu_\alpha \to \nu_\beta, t\big) - P\big(\overline{\nu}_\alpha \to \overline{\nu}_\beta, t\big)\,. \tag{3.31}$$

From CPT invariance we obtain $\Delta P_{\alpha\beta} = -\Delta P_{\beta\alpha}$. Using eq. 3.24, we can write

$$\begin{aligned}\Delta P_{e\mu} = \Delta P_{\mu\tau} = \Delta P_{\tau e} =& 4 s_{12} c_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin\delta \\ & \times \left[\sin\left(\frac{\Delta m_{12}^2}{2E}t\right) + \sin\left(\frac{\Delta m_{23}^2}{2E}t\right) + \sin\left(\frac{\Delta m_{31}^2}{2E}t\right)\right].\end{aligned} \tag{3.32}$$

This expression vanishes for $\delta = 0$. It also vanishes if any of the mixing angles θ_{12}, θ_{13} or θ_{23} is 0° or 90°. Since the mass squared differences satisfy $\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = 0$, the CP-odd asymmetry vanishes if even one of Δm_{ij}^2 is zero. The experimental observation of CP violation in neutrino oscillations is a very difficult task. The CP-odd probability asymmetry is suppressed if any one of the three lepton mixing angles is small, which is the case for θ_{13}. Current neutrino parameters do not exclude the possibility of observation of CP violation effects in future neutrino experiments.

3.1.3 Neutrinos masses in the Standard Model

Remembering the development of chapter 2, in the SM all quarks and charged fermions get their masses through the Yukawa couplings with the Higgs field:

$$\mathcal{L}_{\text{Yuk}} = -\bar{L}_L H y_e e_R - \bar{Q}_L H y_d d_R - \bar{Q}_L \tilde{H} y_u u_R + \text{ h.c.} \tag{3.33}$$

After electroweak symmetry breaking, the Higgs acquires a vev $\langle H \rangle = v/\sqrt{2}$, and the Yukawa terms yield the mass matrices

$$M_e = y_e \frac{v}{\sqrt{2}}, \quad M_d = y_d \frac{v}{\sqrt{2}}, \quad M_u = y_u \frac{v}{\sqrt{2}}. \tag{3.34}$$

In general, different types of mass terms can be built for fermions. Considering a set of fermions ψ_i, with $i \in \{1, 2, \dots, n\}$, we can write the mass terms with a $n \times n$ matrix. Fermions have a Dirac mass term:

$$\mathcal{L}_D = -\sum_{i,j} \overline{\psi}^i_R M^D_{ij} \psi^j_L + \text{h.c.}, \tag{3.35}$$

where we have both left and right components of fermions. Right-handed neutrinos can be added to the SM particle content, producing the gauge-invariant, renormalizable Yukawa term $y_\nu \overline{L} \tilde{H} \nu_R + \text{h.c.}$, but to accommodate the $O(0.1)$ eV neutrino mass scale, the yukawa coupling y_ν would be of the order of 10^{-13}. Therefore, with this model, one needs a set of tiny dimensionless parameters, which are six or seven orders of magnitude smaller than the next smallest Yukawa coupling constant.

The Majorana mass term can be built using the charge conjugated field, in which case the Majorana mass term has the form

$$\mathcal{L}_M = -\frac{1}{2} \sum_{i,j} \overline{\psi}^i_L M^M_{ij} \psi^{j,c}_L + \text{h.c.}, \tag{3.36}$$

where we note that the mass term can be built for right handed neutrinos in an analogous way. Both Majorana and Dirac mass terms can coexist in the Lagrangian.

Neutrinos are massless in the context of the SM. Therefore, we cannot have Dirac masses since there are no right-handed neutrinos ν_R in the Standard Model. The reason why neutrinos cannot have Majorana masses in the SM is more subtle. The Majorana mass term for neutrinos should be of the form $\overline{\nu}_L \overline{\nu}^c_L$. ν_L has weak isospin projection $I_3 = 1/2$, and the Majorana mass term has $I_3 = 1$, meaning that it is a component of the isotriplet operator $L^T_L C i\tau_2 \boldsymbol{\tau} L_L \sim (3, -2)$. To introduce the Majorana mass in a gauge invariant way to preserve the renormalizability of the standard model, we would need an isotriplet Higgs field $\Delta \sim (3,2)$. If one introduces the previous field in the Lagrangian, its electrically neutral component would develop a VEV, causing that a Majorana neutrino mass is generated. However, such field does not exist in the standard model. It is possible to build a composite triplet Higgs operator out of two Higgs doublets, which is the matter of the following section.

3.1.4 Dimension-5 Weinberg operator

Following the discussion of the preceding section, we identify that the operator $H^T i\tau_2 \boldsymbol{\tau} H$ possesses the correct quantum numbers required for constructing the triplet Higgs operator. The term $(L_L^T C i\tau_2 \boldsymbol{\tau} L_L)(H^T i\tau_2 \boldsymbol{\tau} H)$ has dimension 5, preventing its entry into the Lagrangian of a renormalizable model at the fundamental level. However, it could be introduced as an effective operator at a higher loop level.

In the context of treating the Standard Model as an effective field theory, one assumes the adherence of the SM gauge symmetry $G_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$, with the fields heavier than the EWSB scale having neglected dynamics. Within this framework, certain heavy BSM fields may mediate interactions absent in the SM alone, provided each interaction is Lorentz invariant and upholds the gauge symmetry G_{SM}. This consideration necessitates the inclusion of higher-dimensional, non-renormalizable operators. One such term, introduced by Weinberg in 1979, stands out as a unique dimension-five operator capable of generating Majorana neutrino masses[65]. This operator, known as the "dimension-5 Weinberg operator" is the lowest-dimensional, non-renormalizable operator built from standard model fields that is invariant under G_{SM}. It can be succinctly represented as:

$$\mathcal{O}_W = \frac{C}{\Lambda} \overline{L}^c H H L, \tag{3.37}$$

where Λ denotes an effective scale, C represents a dimensionless coefficient, and L and H refer to the lepton and Higgs isodoublets, respectively. In this representation, we have omitted the contraction of Lorentz indices as well as SU(2) indices.

Within the SM, both lepton and baryon numbers are conserved at the perturbative level, owing to the accidental symmetries inherent in the Lagrangian. These symmetries emerge as direct consequences of the particle content, gauge invariance, renormalizability, and Lorentz invariance of the model. The Weinberg operator, being a dimension-5 operator, introduces a violation of lepton number conservation. This violation is considered effective and is expected to be suppressed by the scale Λ at which lepton number symmetry breaks down. Such a description provides an effective theory that encompasses various underlying models. Following spontaneous symmetry breaking, when the Higgs field acquires a non-zero vacuum expectation value $\langle H \rangle = v/\sqrt{2}$, neutrinos gain Majorana masses, expressed as:

$$m = \frac{C}{\Lambda} v^2. \tag{3.38}$$

In the scenario where the scale of new physics, Λ, is large, the neutrino mass naturally becomes small, leading to the seesaw mechanism. Given that terms in the Lagrangian with a dimension larger than five are non-renormalizable, the Weinberg operator is considered effective at energies beyond the scale of Λ.

The Weinberg operator can be generalized to the set

$$\mathcal{O}_W'^{\cdots\prime} \sim LLHH(H^\dagger H)^n, \tag{3.39}$$

where the number of primes equals n. In this case, one obtains a more powerful suppression

$$m \sim v\epsilon^{2n+1} \tag{3.40}$$

as n increases. If one wants to derive, from an underlying renormalizable or UV complete theory, one of the Weinberg-type operators as the leading contributions to neutrino mass, the operator needs to be "opened up". Depending on which operator dominates and how it is opened up, one determines the type of theory is obtained. Some possible choices are[66]:

1. Open up $\mathcal{O}_W$ at tree-level using only exotic massive fermions and scalars as the new physics.
2. Open up $\mathcal{O}_W$ at j-loop level using heavy exotics only.
3. Open up$\mathcal{O}_W$ at j-loop level using both light SM particles and heavy exotics.
4. Open up $\mathcal{O}_W'^{\cdots\prime}$ at tree-level using heavy exotics only.
5. Open up $\mathcal{O}_W'^{\cdots\prime}$ at j-loop level using heavy exotics only.
6. Open up $\mathcal{O}_W'^{\cdots\prime}$ at j-loop level using both light SM particles and heavy exotics.

The first option gives rise to the three distinct types of seesaw mechanisms. These mechanisms are explored further in the following section. The other scenarios lead to different kinds of radiative models. We will study one of such models known as the scotogenic in a subsequent section

3.1.5 The seesaw mechanisms

All Majorana neutrino mass models ultimately reduce to the Weinberg operator or equivalent formulations. There are only three established mechanisms for generating this operator at the tree level, and they collectively fall under the widely recognized category of seesaw mechanisms. The seesaw mechanisms operate on the principle of inducing the operator $\mathcal{O}_W$ by exchanging heavy particles. To implement the seesaw mechanism, we extend the Standard Model by introducing the right-handed neutrino ν_R, which is a singlet with respect to all SM gauge groups. Being electroweak singlets, right-handed neutrinos can possess Majorana mass terms that are invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Additionally, they do not contribute to the electroweak anomaly, and their number is not constrained by the requirement of anomaly cancellation. The quantity of right-handed neutrinos is not bound to coincide with the number of generations in the SM. However, there are astrophysical and cosmological constraints on their number, which depend on their masses and mixing parameters[67, 68].

Majorana mass terms for ν_R are allowed without any restrictions. However, for the left-handed neutrinos, Majorana mass terms will remain forbidden since they violate gauge invariance. For simplicity, we consider only one neutrino generation. The corresponding Lagrangian has the form[69]:

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} m_L \overline{\nu}_L \nu_L^c - \frac{1}{2} m_R \overline{\nu}_R^c \nu_R - m_D \overline{\nu}_L \nu_R + \mathrm{h.c.}, \tag{3.41}$$

where, m_L, m_D, and m_R are real parameters. In this case, we can group the Dirac and Majorana mass terms in a mass matrix, giving the Lagrangian the following form:

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \overline{n}_L M^{D+M} (n_L)^c + \mathrm{h.c.}, \tag{3.42}$$

where

$$M^{\mathrm{D+M}} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}, \quad n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}. \tag{3.43}$$

The mass matrix $M^{\mathrm{D+M}}$ indicates that there exists a mixing, but this matrix can be diagonalized, leading to separate mass eigenstates that do not mix. We diagonalize the matrix with a transformation $U^T M^{\mathrm{D+M}} U = m_d$, where U is an orthogonal matrix, from which we obtain the mixing angle, given by:

$$\tan 2\theta = \frac{2m_D}{m_R - m_L}. \tag{3.44}$$

The masses of the physical fields are therefore the eigenvalues of $M^{\mathrm{D+M}}$. These eigenvalues are given by:

$$m_{1,2} = \frac{1}{2}\left(m_L + m_R\right) \mp \frac{1}{2}\sqrt{4m_D^2 + (m_R - m_L)^2}. \tag{3.45}$$

Considering the mixing, the seesaw mechanism can be introduced in the SM to explain the neutrino mass generation. In the SM, there is no left-handed Majorana mass term, since a left-handed Majorana term would violate gauge charges, so we can assume $m_L = 0$ to respect unbroken symmetries. The right-handed neutrinos are gauge singlets, and we can introduce their Majorana mass term without further problems. The Dirac component comes from a standard Yukawa interaction. Lepton number is violated at a scale much larger than the electroweak scale, which implies $m_R \gg m_D$, which is the main assumption in the seesaw mechanism. Under these assumptions, the eigenvalues take the form:

$$m_1 \simeq -\frac{m_D^2}{m_R}, \quad m_2 \simeq m_R. \tag{3.46}$$

The relation between the physical and the original states is given by

$$\begin{aligned} \nu_L &= i\nu_{1L} + \frac{m_D}{m_R}\nu_{2L}, \\ (\nu_R)^c &= -i\frac{m_D}{m_R}\nu_{1L} + \nu_{2L}. \end{aligned} \tag{3.47}$$

In this simplified case, we note that active neutrinos acquire naturally small masses suppressed by m_R, which could be arbitrarily large.

We now consider the general case of n generations. Now, the Dirac and Majorana seesaw matrix has the form:

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}, \quad n_L = \begin{pmatrix} v_L \\ (v_R)^c \end{pmatrix}. \tag{3.48}$$

This matrix can be diagonalized with a $2n \times 2n$ unitary matrix V such that $V^T M V = m$. The matrix V can be of the form:

$$V = \begin{pmatrix} 1 & \rho \\ -\rho^\dagger & 1 \end{pmatrix}, \quad V^\dagger V = 1 + \mathcal{O}\left(\rho^2\right), \tag{3.49}$$

where its elements are $n \times n$ matrices, and ρ is treated as a perturbation. For simplicity, we can neglect possible CP violation in the leptonic sector, and consider m_D and M_R to be real matrices. The matrix ρ can also be taken as real. Considering the previous assumptions, upon block-diagonalization of the matrix M, the matrix m takes the form:

$$m \simeq \begin{pmatrix} -m_D M_R^{-1} m_D^T & 0 \\ 0 & M_R \end{pmatrix}. \tag{3.50}$$

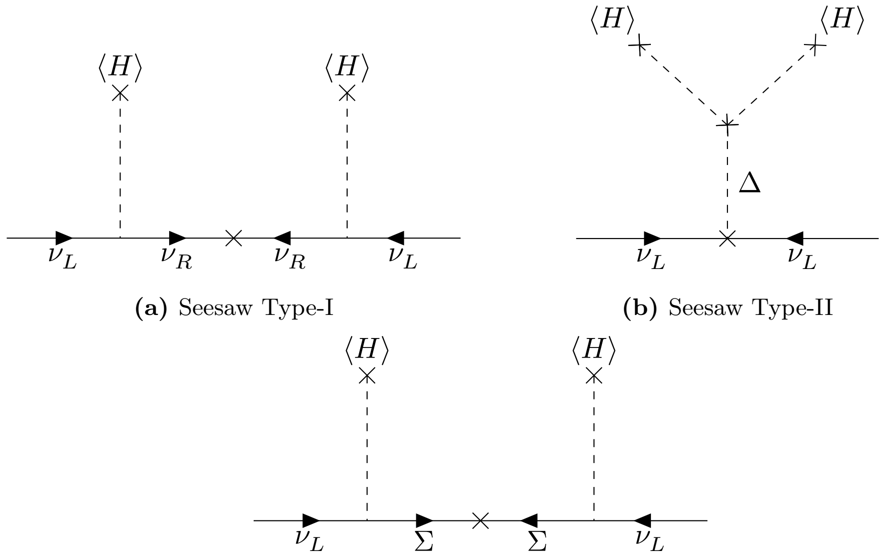
As a result, we observe that the left-handed Majorana matrix is given by:

$$m_L = -m_D M_R^{-1} m_D^T. \tag{3.51}$$

The diagonalization of the effective mass matrix m_L yields n light Majorana neutrinos, composed primarily by the active neutrinos ν_L, with a very small contribution of the sterile neutrinos ν_R. On the other hand, diagonalization of M_R produces n heavy Majorana neutrinos which are mainly composed of ν_R. This tells us that the heavier ν_R is, the lighter ν_L will be. The masses of active neutrinos are of the order of m_D^2/M_R. The case we just considered is known as Type-I seesaw mechanism[70]. This mechanism is illustrated in Fig. 3.1a. Interestingly, considering the largest Dirac mass eigenvalue of the order of the electroweak scale, around 200 GeV, and the right handed scale as $M_R \sim 10^{15}$ GeV, one obtains the mass of the heaviest light neutrino m_ν to be around $(10^{-2} - 10^{-1})$ eV, which is of the same order of magnitude for the neutrino oscillation solution of the atmospheric neutrino anomaly[71].

The Type III seesaw mechanism[72] is analogous to the Type I seesaw, but it involves the introduction of an SU(2) triplet fermion Σ with hypercharge zero, instead of the right-handed neutrino ν_R. The diagram corresponding to the Type III seesaw is shown in Fig. 3.1c. In this scenario, the electrically neutral component of Σ can be regarded as the right-handed neutrino. It can possess a Majorana mass, and when combined with the left-handed neutrino, it allows for a Dirac mass term.

On the other hand, the Type II seesaw mechanism[73, 74] involves the inclusion of a scalar SU(2) triplet (Δ) with hypercharge 1 to the Standard Model. In this case, there is no additional fermion introduced for the neutrino to mix with, leading to the absence of a Dirac mass term. This model is the unique theory obtained from Yukawa coupling the fermion bilinear LL to Δ which in turn couples to $H^\dagger H^\dagger$, a cubic interaction term in the scalar potential. In this case, the The seesaw effect is obtained upon requiring a positive quadratic term for the triplet in the scalar potential, that on its own would cause the triplet's VEV to vanish, but which in combination with the cubic term induces a small VEV. Its corresponding diagram is shown in Fig. 3.1b.



(a) Seesaw Type-I

(b) Seesaw Type-II

(c) Seesaw Type-III

Figure 3.1: Seesaw mechanisms diagrams.

The three seesaw mechanisms discussed represent the simplest realizations of neutrino Majorana masses with an extension of the SM that involves the addition of a single BSM field. In the tree level seesaws, the neutrino mass scale is roughly given by $\mu_\nu = \langle H \rangle / \Lambda$, with $\langle H \rangle$ the VEV of the Higgs.

3.2 The scotogenic model

As discussed earlier in this chapter, the minimal introduction of sterile neutrinos into the SM would necessitate extremely small Yukawa couplings. While this scenario is theoretically possible, it has proven unsatisfactory to many researchers. The observed tiny neutrino masses strongly suggest an alternative mechanism for neutrino mass generation that can rationally account for their small values. This has led to the development of numerous models explaining Majorana masses, with fewer addressing Dirac masses. In this section, we delve into the scotogenic model, a foundational framework that has inspired many models explaining both tiny Majorana and Dirac neutrino masses.

The scotogenic model, initially proposed by Ernest Ma in 2006[21], offers an intriguing framework where particles responsible for generating neutrino masses also serve as potential dark matter candidates. This model employs a one-loop mechanism for neutrino mass generation, utilizing interactions between the dark sector and neutrinos to induce small neutrino masses. Constituting a minimal extension of the Standard Model, the scotogenic model introduces three neutral fermions $N_{1,2,3}$ and a single SU(2) doublet $\eta = (\eta^+, \eta^0)^T$. Both N_i and η are odd under an additional Z_2 symmetry. The inclusion of this symmetry ensures that the η doublet cannot acquire a nonzero VEV, making it a suitable candidate for dark matter. Simultaneously, it prevents the existence of tree-level neutrino masses, preserving the viability of neutrinos as massless particles at the tree level. However, a loop-level Majorana mass for neutrinos can still arise within the scotogenic model due to the interactions of the dark sector with neutrinos.

The additional terms in the Lagrangian responsible for neutrino mass in the scotogenic model are given by:

$$\mathcal{L}_Y = -h_{ij}\overline{L}_{iL}\eta N_j - \frac{1}{2}M_{ij}\overline{N}_i N_j^c + \text{h.c.}, \tag{3.52}$$

$$\mathcal{L}_V = \lambda_3 \left(H^\dagger H\right)\left(\eta^\dagger \eta\right) + \lambda_4 \left(H^\dagger \eta\right)\left(\eta^\dagger H\right) + \frac{1}{2}\lambda_5 \left[\left(H^\dagger \eta\right)^2 + \left(\eta^\dagger H\right)^2\right], \tag{3.53}$$

where all λ_i can be chosen to be real without any loss of generality. After ϕ^0 acquires a nonzero VEV (v) and using $\eta^0 = \eta_R + i\eta_I$, the λ_5 term can be expressed as:

$$\frac{1}{2}\gamma_5 v^2 \left(\eta_R^2 - \eta_I^2\right). \tag{3.54}$$

This particular term induces a mass splitting between the real (η_R) and imaginary (η_I) components of the dark scalar doublet, leading to distinct masses for these two states. The diagram of the radiative generation of neutrino mass, is shown in Fig. 3.2

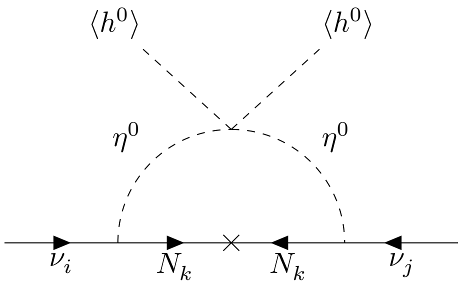


Figure 3.2: One-loop generation of neutrino mass.

The scotogenic mass term for the previous diagram is given by the integral of the form[75]:

$$\begin{aligned} I_{ij} &= \sum_k \frac{h_{ik}h_{jk}}{2}\int \frac{d^4k}{(2\pi)^4} i\left(\not{k} + m_{N_k}\right)\left[\frac{1}{\left(k^2 - m_{N_k}^2\right)\left(k^2 - m_R^2\right)} - \frac{1}{\left(k^2 - m_{N_k}^2\right)\left(k^2 - m_I^2\right)}\right] \\ &= \sum_k \frac{h_{ik}h_{jk}}{2}\int \frac{d^4k}{(2\pi)^4} i\left(\not{k} + m_{N_k}\right)\left[\frac{m_R^2 - m_I^2}{\left(k^2 - m_{N_k}^2\right)\left(k^2 - m_R^2\right)\left(k^2 - m_I^2\right)}\right]. \end{aligned} \tag{3.55}$$

To evaluate this integral, we utilize a generalization of Feynman's formula for n propagators[76] (labeled A_n), each repeated m_i times in the denominator:

$$\frac{1}{A_1^{m_1}A_2^{m_2}\dots A_n^{m_n}}=\int_0^1 dx_1\dots dx_n\delta\left(\sum x_i-1\right)\frac{\Pi x_i^{m_i-1}}{\left(\sum x_iA_i\right)^{\sum m_i}}\frac{\Gamma\left(\sum m_i\right)}{\Gamma(m1)\dots\Gamma\left(m_n\right)}. \tag{3.56}$$

Using this formula, the integral can be expressed as follows:

$$\begin{aligned}I_{ij}=&\sum_k\frac{ih_{ik}h_{jk}\left(m_R^2-m_I^2\right)}{2}\int\frac{d^4k}{(2\pi)^4}\left(\not{k}+m_{N_k}\right)\times\\&\left[\int dxdydz\frac{\delta\left(x+y+z-1\right)\Gamma(3)}{\left[x\left(k^2-m_{N_k}^2\right)+y\left(k^2-m_R^2\right)+z\left(k^2-m_I^2\right)\right]^3}\right]\\=&\sum_k\frac{ih_{ik}h_{jk}\left(m_R^2-m_I^2\right)}{2}\int\frac{d^4k}{(2\pi)^4}\left(\not{k}+m_{N_k}\right)\times\\&\int_0^1 dx\int_0^{1-x}dy\frac{\Gamma(3)}{\left[x(\left(k^2-m_{N_k}^2\right))+y\left(k^2-m_R^2\right)+(1-x-y)\left(k^2-m_I^2\right)\right]^3}\end{aligned} \tag{3.57}$$

Next, we rewrite the denominator as $\Delta=x(m_{N_k}^2-m_I^2)+y(m_R^2-m_I^2)+m_I^2$, resulting in the expression:

$$I_{ij}=\sum_k ih_{ik}h_{jk}\left(m_R^2-m_I^2\right)\int_0^1 dx\int_0^{1-x}dy\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}+m_{N_k}}{\left(k^2-\Delta\right)^3} \tag{3.58}$$

As a next step, the integral over $\not{k}$ is shown to vanish due to symmetry. By using a Wick rotation ($k_0=il_0$ so that $k^2\to-l^2$) the integral over k can be performed in a four-dimensional spherical space, leading to:

$$\begin{aligned}I_{ij}&=\sum_k\frac{h_{ik}h_{jk}m_{N_k}\left(m_R^2-m_I^2\right)}{(2\pi)^4}\int_0^1 dx\int_0^{1-x}dy\int\frac{l^3dld\Omega}{\Delta^3\left(1+\frac{l^2}{\Delta}\right)^3}\\&=\sum_k\frac{h_{ik}h_{jk}m_{N_k}\left(m_R^2-m_I^2\right)}{(2\pi)^4}\int_0^1 dx\int_0^{1-x}dy\int\frac{2\pi^2l^3dl}{\Delta^3\left(1+\frac{l^2}{\Delta}\right)^3}\\&=\sum_k\frac{h_{ik}h_{jk}m_{N_k}\left(m_R^2-m_I^2\right)}{32\pi^2}\int_0^1 dx\int_0^{1-x}dy\frac{1}{x\left(m_{N_k}^2-m_I^2\right)+y\left(m_R^2-m_I^2\right)+m_I^2}\\&=\sum_k\frac{h_{ik}h_{jk}m_{N_k}}{32\pi^2}\int_0^1 dx\log\left(\frac{x\left(m_{N_k}^2-m_R^2\right)+m_R^2}{x\left(m_{N_k}^2-m_I^2\right)+m_I^2}\right)\\&=\sum_k\frac{h_{ik}h_{jk}m_{N_k}}{32\pi^2}\left(\frac{m_{N_k}^2\log m_{N_k}^2-m_R^2\log m_R^2}{m_{N_k}^2-m_R^2}-\frac{m_{N_k}^2\log m_{N_k}^2-m_I^2\log m_I^2}{m_{N_k}^2-m_I^2}\right)\\&=\sum_k\frac{h_{ik}h_{jk}m_{N_k}}{32\pi^2}\left(\frac{m_R^2}{m_R^2-m_{N_k}^2}\log\left(\frac{m_R^2}{m_{N_k}^2}\right)-\frac{m_I^2}{m_I^2-m_{N_k}^2}\log\left(\frac{m_I^2}{m_{N_k}^2}\right)\right)\end{aligned} \tag{3.59}$$

Taking different mass limits for m_R, m_I, and M_{N_k}, this formula can produce both a seesaw and an inverse seesaw-like relation between the mass of M_{N_k} and that of the neutrino masses. In the following chapter, we will further develop an extension to the scotogenic model.

3.2.1 Additional radiative schemes for neutrino masses

As was mentioned in sec. 3.1.4, there are possible radiative neutrino mass models which derive from the Weinberg operator and adopt the number of loops as the primary consideration. At j-loop order, neutrino masses are typically given by

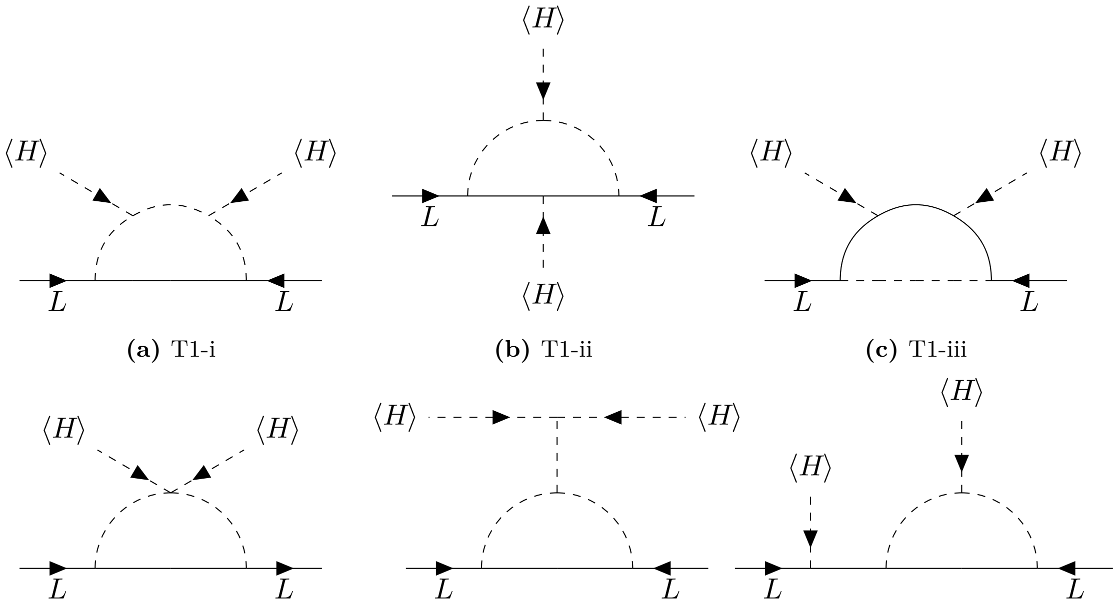
$$m_\nu \sim C \left(\frac{1}{16\pi^2}\right)^j \frac{v^2}{\Lambda} \tag{3.60}$$

for the $\mathcal{O}_W$ associated options 2 and 3 previously introduced. For the $\mathcal{O}'_W$ cases of options 5 and 6, are typically given by

$$m_\nu \sim C \left(\frac{1}{16\pi^2}\right)^j \frac{v^4}{\Lambda^3}, \tag{3.61}$$

where $v = \sqrt{2}\langle H\rangle \simeq 100$ GeV. All coupling constants, and for some models also certain mass-scale ratios, are absorbed in the dimensionless coefficient C.

Most of the radiative neutrino mass models generate neutrino mass at 1, 2 and 3-loop level. In [77] 12 topologies were identified which contribute to neutrino mass. Some of the topologies cannot be realized in a renormalizable theory or require counter-terms to absorb divergences. There are six topologies which generate neutrino mass via a 1-loop diagram, which we label as T1-i, T1-ii, T1-iii, T3, T4-2-i and T4-3-i, depicted in Fig. 3.3:



(a) T1-i **(b)** T1-ii **(c)** T1-iii

(d) T3 **(e)** T4-2-i **(f)** T4-3-i

Figure 3.3: Feynman diagram topologies for 1-loop radiative neutrino mass generation with the Weinberg operator.

These are also called the UV completions of the Weinberg operator at 1-loop. Depending on the particle content, the topologies do not rely on any additional symmetry. The topology T3 is the only one with a quartic scalar interaction. The scotogenic model was its first realization. Several variants of the scotogenic model have been proposed in the literature, which include the addition of new fermions, additional scalars, higher SU(2) representations, extended discrete symmetries, etc[78–83]. Many other models have been built using other topologies to generate neutrino masses. For example, in the reference [84] a topology $T1 - i$ was used. Among the models based on the topology T1-ii, there are four possible operators which models are based on. The topology T1-iii appears in the sypersymmetrized version of the scotogenic model[85].

Models with additional scalar fields contribute to neutrino mass via their vacuum expectation value in contrast to being a propagating degree of freedom in the loop. In the following chapter we introduce a variant of the scotogenic model with additional VEV insertions. Our model extends the symmetry sector with a gauged $\mathrm{U}(1)_L$ symmetry.

4

An Extension of the Scotogenic Model with Gauged Lepton Number

Contents

In the preceding chapter, we delved into the Scotogenic model, an elegant framework representing the simplest model of radiative neutrino masses. This model not only unifies the generation of SM neutrino masses with dark matter but also introduces intriguing features. In this chapter, we extend the Scotogenic model by elevating lepton number conservation to the status of an $\mathrm{U}(1)_L$ gauge symmetry spontaneously broken by three units.

The Scotogenic model incorporates a second SU(2) scalar doublet η and at least two right-handed neutrinos N_i, all transforming with odd parity under an exact $\mathbb{Z}_2$ symmetry. This symmetry serves a dual purpose: ensuring the stability of the lightest $\mathbb{Z}_2$-odd state, a potential dark matter candidate if electrically neutral, and preventing the η doublet from acquiring a non-zero VEV, thereby avoiding tree-level neutrino masses. However, justifying the theoretical basis for this $\mathbb{Z}_2$ symmetry imposition is not straightforward, as higher $\mathbb{Z}_n$ symmetries could play a similar role. Additionally, theoretical arguments suggest that discrete symmetries should have a dynamical origin[86], such as a global U(1) symmetry, preventing issues like the formation of domain walls[87].

The Scotogenic model has undergone various extensions[88–90], all leveraging the involvement of new particles from the dark sector within loops to induce neutrino masses. Extensions encompass scenarios where the dark matter symmetry aligns with a U(1) gauge symmetry[91] or originates from lepton number conservation[92]. Additionally, extensions have explored SU(5) gauge interactions[93], SU(5) unification[94], the inverse seesaw mechanism[95], and the origin of lighter generations of lepton and quark masses[96].

Motivated by the non-observation of neutrinoless double beta decay and the conservation of total lepton number as a robust symmetry in nature, we introduce an extension of the Scotogenic model. This extension elevates lepton number to the status of an $\mathrm{U}(1)_L$ gauge symmetry, which is spontaneously broken by three units. Consequently, a new set of leptons with appropriate quantum numbers is introduced to cancel anomalies in the model.

4.1 Gauged Lepton number

In the minimal Standard Model, both baryon number (B) and lepton number (L) emerge as accidental global symmetries. However, as demonstrated in Chapter 2, these symmetries are anomalous in the SM, necessitating specific conditions for the gauging of either symmetry. Notably, $B - L$ can be gauged by introducing a singlet right-handed neutrino (ν_R) per family. When allowing B and L to vary among different families, the requirement for an anomaly-free

$\mathrm{U}(1)_X$ gauge symmetry is expressed as[97]:

$$\sum_{i=1}^{3}(3n_i + n_i') = 0, \tag{4.1}$$

where n_i and n_i' denote the $\mathrm{U}(1)_X$ values for each quark and lepton family.

The independent gauging of B and L[98] becomes feasible with the introduction of new fermions. Given that ν_R is a novel addition, its assignment under $\mathrm{U}(1)_L$ is not constrained to match that of ν_L, which must align with charged leptons. This introduces the interesting possibility that neutrinos naturally emerge as light Dirac fermions. To prevent ν_R from acquiring a Majorana mass, $\mathrm{U}(1)_L$ must remain unbroken by any single scalar with a $\mathrm{U}(1)_L$ charge double that of ν_R.

A $\mathrm{U}(1)$ symmetry only admits $\mathbb{Z}_n$ subgroups, where in this case we denote as $\mathbb{Z}_n$ the cyclic group of n elements. This group is characterized by the following property: if g is an element of the group different from the identity, then $g^{n+1} = g$. This type of groups only admit one-dimensional irreducible representations, which are usually represented by $\omega = \exp{(2\pi l)}/n$, with $\omega^n = 1$. There are two possible cases depending on how lepton number ir broken to a $\mathbb{Z}_n$ subgroup:

- If $\mathrm{U}(1)_L \to \mathbb{Z}_n = \mathbb{Z}_{2k+1}$, with k an integer such that $k \geq 1$, then neutrinos are Dirac particles.

- If $\mathrm{U}(1)_L \to \mathbb{Z}_n = \mathbb{Z}_{2k}$, with k an integer such that $k \geq 1$, then neutrinos could be Dirac or Majorana particles.

In the latter case, there is possible to separate two possible scenarios, depending on the charges of neutrinos under the unbroken $\mathbb{Z}_{2k}$ symmetry[99]:

- If $\nu \sim \omega^k$ under $\mathbb{Z}_{2k}$, then neutrinos are Majorana particles.

- If $\nu \nsim \omega^k$ under $\mathbb{Z}_{2k}$, then neutrinos are Dirac particles.

Therefore, from a symmetry point of view, there are more possibilities that lead to Dirac neutrinos than Majorana. This is in contrast from the common idea thar neutrinos should be Majorana particles due to their complete charge neutrality.

The basic setup provided by the standard model is listed in table 4.1

Field	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	$U(1)_L$	ω^{2L}
Q_{iL}	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0	1
u_{iR}	$\mathbf{3}$	$\mathbf{1}$	$2/3$	0	1
d_{iR}	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	0	1
L_{iL}	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	1	ω^2
e_{iR}	$\mathbf{1}$	$\mathbf{1}$	-1	1	ω^2
H	$\mathbf{1}$	$\mathbf{2}$	$1/2$	0	1

Table 4.1: Standard Model fermions and scalars and their gauge transformation properties and the global L.

The remnant symmetry of our model will be $\mathbb{Z}_6$, which allows the neutrinos of our model to be Dirac particles as expected. The additional particles of our model are listed in table 4.2. Some additions to the matter content are needed to ensure anomaly cancellation. We have included three new charged fields under L: in the scalar sector, a doublet η, and the singlets ϕ and σ. In the fermionic field, we have added the right-handed neutrinos ν_{aR} and ν_{3R}, the gauge singlets S_{iL} and S_{iR}, and the fields n_R and n_L. Note that, in contrast with other related works where the chosen lepton number is integer for the new scalar fields, here we have decided to assign half-integer lepton number to η and σ.

Field	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	$U(1)_L$	ω^{2L}
ν_{aR}	$\mathbf{1}$	$\mathbf{1}$	0	4	ω^2
ν_{3R}	$\mathbf{1}$	$\mathbf{1}$	0	-5	ω^2
L'_L	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$l-3$	ω^{2l}
e'_R	$\mathbf{1}$	$\mathbf{1}$	-1	$l-3$	ω^{2l}
n_R	$\mathbf{1}$	$\mathbf{1}$	0	$l-3$	ω^{2l}
L''_R	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	l	ω^{2l}
e''_L	$\mathbf{1}$	$\mathbf{1}$	-1	l	ω^{2l}
n_L	$\mathbf{1}$	$\mathbf{1}$	0	l	ω^{2l}
S_{iL}	$\mathbf{1}$	$\mathbf{1}$	0	$1/2$	ω
S_{iR}	$\mathbf{1}$	$\mathbf{1}$	0	$1/2$	ω
ϕ	$\mathbf{1}$	$\mathbf{1}$	0	3	1
η	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$-1/2$	ω^5
σ	$\mathbf{1}$	$\mathbf{1}$	0	$-7/2$	ω^5

Table 4.2: Our SM extension: new fields and their symmetry properties.

We note the first set of new fermions, which is a sequential generation of fermions with lepton number $L = l - 3$. The second set of fermions with opposite chirality has lepton number l. This is necessary, since the difference of 3 is required by anomaly cancellation. We show how the chosen model is free of anomalies:

$$
\begin{aligned}
[\mathrm{SU}(2)_W]^2\,\mathrm{U}(1)_L &: 3(3)(1/6) + (1/2)[-3] = 0, \\
[\mathrm{U}(1)_Y]^2\,\mathrm{U}(1)_L &: 3[2(-1/2)^2 - (-1)^2] + 2(-1/2)^2(l-3-l) + (-1)^2 l - (-1)^2(l-3) = 0, \\
\mathrm{U}(1)_Y[\mathrm{U}(1)_L]^2 &: 3[2(-1/2) - (-1)] + 2(-1/2)[(l-3)^2 - l^2] - l^2 - (-1)(l-3)^2 = 0, \\
[\mathrm{U}(1)_L]^3 &: 3[2-1] - 2(4)^3 - (-5)^3 + 2[(l-3)^3 - l^3] + l^3 - (l-3)^3 \\
&\quad + [l^3 - (l-3)^3] + [(1/2)^3 - (1/2)^3] = 0, \\
\mathrm{U}(1)_L &: 3[2-1] - 2(4) - (-5) + 2[(l-3) - l] + l - (l-3) + [l - (l-3)] \\
&\quad + [(1/2) - (1/2)] = 0.
\end{aligned}
\tag{4.2}
$$

The singlet scalar ϕ of our model is the responsible for the spontaneous breaking of lepton number gauge symmetry. Note that because of the selected L charges of the right-handed neutrinos, the SM Higgs H does not connect ν_L with ν_R. Therefore, the neutrinos do not get mass from the standard electroweak symmetry breaking.

4.1.1 Scalar spectrum

Given the fields and symmetries of our model, we can construct the simplest scalar potential, which is given by:

$$
\begin{aligned}
V \;=\; & \sum_{s=H,\phi,\eta,\sigma} [\mu_s^2(s^\dagger s) + \lambda_s (s^\dagger s)^2] + \lambda_{H\eta}(H^\dagger H)(\eta^\dagger \eta) + \lambda'_{H\eta}(H^\dagger \eta)(\eta^\dagger H) \\
& + \lambda_{H\sigma}(H^\dagger H)(\sigma^* \sigma) + \lambda_{H\phi}(H^\dagger H)(\phi^*\phi) + \lambda_{\eta\sigma}(\eta^\dagger\eta)(\sigma^*\sigma) + \lambda_{\eta\phi}(\eta^\dagger\eta)(\phi^*\phi) \\
& + \lambda_{\sigma\phi}(\sigma^*\sigma)(\phi^*\phi) + \frac{\mu_4}{\sqrt{2}}(\eta^\dagger H \sigma\phi + \mathrm{h.c})\,,
\end{aligned}
\tag{4.3}
$$

where the mass parameter μ_4 is assumed real for simplicity. From V it can be noted that ϕ breaks $\mathrm{U}(1)_L$ by three units. After EWSB there is a remnant gauge discrete symmetry $\mathbb{Z}_6$ with charges ω^{2L}, where $\omega = \exp(2\pi i/3)$. Both the fields H and ϕ will acquire a vev. The CP-even field of the neutral component of H acquires a mass $m_{h^0}^2 = 2\lambda_H v^2$, and is identified with the Higgs boson, which was observed in 2012 at LHC with a mass of 125 GeV. The remaining components of H are absorbed by the gauge sector via the Higgs mechanism, generating the bosons $W^\pm$ and Z.

The dark sector will be formed by the rest of the scalar fields which are odd under matter parity, and hence do not acquire a vev. The scalar doublets can be expanded into components according to

$$H = \begin{pmatrix} h^+ \\ \frac{v+s+ia}{\sqrt{2}} \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^+ \\ \frac{\eta_R + i\eta_i}{\sqrt{2}} \end{pmatrix}, \tag{4.4}$$

where $v/\sqrt{2}$ represents the vev acquired by the neutral component of H. We have also decomposed η^0 in real and imaginary parts. The field ϕ is written as

$$\phi = \frac{w + \phi_r + i\phi_i}{\sqrt{2}}, \tag{4.5}$$

where $\langle\phi\rangle = w/\sqrt{2}$ represents the vev acquired by the real component of ϕ after lepton number is broken, and is the only new dimensional scale introduced, with all of the other parameters being dimensionless couplings.

The expectation values can be determined from the minimization condition

$$\left.\frac{\partial V}{\partial s}\right|_{\substack{H=\langle H\rangle \\ \phi=\langle\phi\rangle}} = \left.\frac{\partial V}{\partial \phi_r}\right|_{\substack{H=\langle H\rangle \\ \phi=\langle\phi\rangle}} = 0. \tag{4.6}$$

The minimization condition of the scalar potential yields the following relations:

$$\begin{aligned} \mu_H^2 &= -\tfrac{1}{2}\left(2v^2\lambda_H + w^2\lambda_{H\phi}\right), \\ \mu_\phi^2 &= -\tfrac{1}{2}\left(v^2\lambda_{H\phi} + 2w^2\lambda_\phi\right). \end{aligned} \tag{4.7}$$

The first component of the scalar doublet η corresponds to a massive charged scalar field, $\eta^\pm$, whose mass is

$$m_{\eta^\pm}^2 = \mu_\eta^2 + \frac{\lambda_{H\eta} v^2}{2} + \frac{\lambda_{\eta\phi} w^2}{2}. \tag{4.8}$$

The spontaneous symmetry breaking induces a mixing between the fields s and ϕ_r, given by the matrix

$$M_{s,\phi_r}^2 = \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H\phi} vw \\ \lambda_{H\phi} vw & 2\lambda_\phi w^2 \end{pmatrix}. \tag{4.9}$$

The previous matrix can be diagonalized via

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} s \\ \phi_r \end{pmatrix}, \tag{4.10}$$

with

$$\tan 2\theta = \frac{vw\lambda_{H\phi}}{w^2\lambda_\phi - v^2\lambda_H}. \tag{4.11}$$

This is the mixing angle that parametrizes the mixing between the real singlet components of

H and ϕ. This mixing is a consequence of the presence of the 'Higgs portal' coupling $\lambda_{H\phi}$. The fields φ_1 and φ_2 are the mass eigenstates. Their masses are found upon diagonalizing the previous matrix. The resulting mass eigenstates are

$$m_{\varphi_1,\varphi_2} = \lambda_H v^2 + \lambda_\phi w^2 \mp \sqrt{\lambda_H^2 v^4 - 2\lambda_H \lambda_\phi v^2 w^2 + \lambda_{H\phi}^2 v^2 w^2 + \lambda_\phi^2 w^4}. \tag{4.12}$$

The coupling $\lambda_{H\phi}$ leads to a tree shift in the Higgs quartic coupling[100], which provides a mechanism to stabilize the vacuum in the presence of the exotic charged leptons with large Yukawa couplings to the Higgs. It has also been shown to be a particularly efficient stabilization mechanism when $m_{\varphi_2} \gg m_{\varphi_1}$, even for small mixing angles[101]. The fields η^0 and σ acquire a vev mix arising from $(\eta^0, \sigma) M_\varphi^2 (\eta^0, \sigma)^\dagger$, where

$$M_\varphi^2 = \frac{1}{2} \begin{pmatrix} 2\mu_\eta^2 + \lambda'_{H\eta} v^2 + \lambda_{H\eta} v^2 + \lambda_{\eta\phi} w^2 & \frac{\mu_4 v w}{\sqrt{2}} \\ \frac{\mu_4 v w}{\sqrt{2}} & 2\mu_\sigma^2 + \lambda_{H\sigma} v^2 + \lambda_{\sigma\phi} w^2 \end{pmatrix}. \tag{4.13}$$

Upon diagonalizing the mass matrix above, we find two complex neutral scalars in the spectrum

$$\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta^0 \\ \sigma \end{pmatrix} \tag{4.14}$$

where

$$\tan(2\theta) = \frac{\sqrt{2}\mu_4 v w}{2(\mu_\eta^2 - \mu_\sigma^2) + v^2(\lambda'_{H\eta} + \lambda_{H\eta} - \lambda_{H\sigma}) + w^2(\lambda_{\eta\phi} - \lambda_{\sigma\phi})} \tag{4.15}$$

The mass eigenvalues of previous states are given by

$$m^2_{\varphi^0_{1,2}} = \frac{1}{4} \Big[2(\mu_\eta^2 + \mu_\sigma^2) + v^2(\lambda'_{H\eta} + \lambda_{H\eta} + \lambda_{H\sigma}) + w^2(\lambda_{\eta\phi} + \lambda_{\sigma\phi}) \tag{4.16}$$

$$- \sqrt{\Big(2(\mu_\eta^2 - \mu_\sigma^2) + v^2(\lambda'_{H\eta} + \lambda_{H\eta} - \lambda_{H\sigma}) + w^2(\lambda_{\eta\phi} - \lambda_{\sigma\phi})\Big)^2 + 2\mu_4^2 v^2 w^2} \Big] \tag{4.17}$$

In Eq. 4.13 the real and imaginary parts of the scalar fields appear together, that is, real and imaginary part are degenerate in mass. That means that, if dark matter is scalar, it is described by a complex field, in contrast with conventional scotogenic scenarios, in which they are nearly degenerate but not exactly so.

4.1.2 Gauge sector

There is a new gauge boson Z_L associated to $\mathrm{U}(1)_L$. We consider the unitary gauge, and use

$$\langle H\rangle = \left(\begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array}\right), \quad \langle\phi\rangle = \frac{w}{\sqrt{2}}. \tag{4.18}$$

The covariant derivative is defined as

$$D_\mu = \partial_\mu + ig\tau_i W_i^\mu + ig' Y B^\mu + ig_L L Z_L^\mu, \tag{4.19}$$

and we also make use of the definition

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_{\mu 1} \mp iW_{\mu 2}\right), \tag{4.20}$$

to study the interactions of Z_L^μ, which are encoded in the following Lagrangian density

$$\begin{aligned}\mathcal{L}_{Z_L} \supset \left(D^\mu H\right)^\dagger\left(D_\mu H\right) + \left(D^\mu\phi\right)^\dagger\left(D_\mu\phi\right) &= \tfrac{1}{8}v^2\left[\left(gW_3^\mu - g'B^\mu\right)^2 + 2g^2 W^{\mu+}W_\mu^-\right] \\ &+ \tfrac{1}{8}\left(36 g_L^2 w^2 Z_L^\mu Z_{L\mu}\right).\end{aligned} \tag{4.21}$$

The previous Lagrangian contains additional terms, such as $\frac{\epsilon}{2}Z_L^{\mu\nu}B_{\mu\nu}$, $\bar{L}_L\gamma^\mu D_\mu L_L$, $\bar{L}'_L\gamma^\mu D_\mu L'_L$, $\bar{L}''_R\gamma^\mu D_\mu L''_R$, where $Z_L^{\mu\nu}$ and $B_{\mu\nu}$ are the $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_Y$ field strength tensors, respectively. There is no $\delta M^2 Z_{L\mu}Z^\mu$ term since ϕ is not charged under the SM, and the Higgs has $L=0$. The left handed SM lepton doublets L_L couple to Z_L, and because of the U(1) kinetic mixing $Z - Z_L$, there is a $B^\mu - Z_L^\mu$ coupling parametrized by ϵ[102]. As it was noted in [103], the strongest bound on Z_L comes from LEP II data[104]:

$$w \geq 1.7 \text{ TeV}, \tag{4.22}$$

which is roughly independent of the g_L value.

The imposition of $\epsilon = 0$ at tree level through symmetries is possible, although in general it is a free parameter of the theory and is additively renormalized by loops of leptons. The inclusion of this term causes that after lepton and electroweak symmetry breaking the $Z - Z_L$ mixing is parametrized by

$$\tan 2\xi = \frac{2M_Z^2 s_w \epsilon\sqrt{1-\epsilon^2}}{M_{Z_L}^2 - M_Z^2(1-\epsilon^2) + M_Z^2 s_w^2\epsilon^2}, \tag{4.23}$$

where ξ is the $Z_L - Z$ mixing angle, M_Z and M_{Z_L} are the masses of Z and Z_L respectively, and s_w is the sine of the weak angle.

In the following calculations, we take $\epsilon = 0$ at tree level and set the scale of $\mathrm{U}(1)_L$ symmetry breaking at its lower bound $w = 1.7$ TeV. With these considerations, we rewrite the following term

$$\frac{1}{8}v^2 \left(gW_3^\mu - g'B^\mu\right)^2 = \frac{g^2+g'^2}{8}v^2\left(\frac{gW_3^\mu - g'B^\mu}{\sqrt{g^2+g'^2}}\right)^2. \tag{4.24}$$

We can reduce this term by noticing that

$$\left(\frac{gW_3^\mu - g'B^\mu}{\sqrt{g^2+g'^2}}\right)^2 = \frac{1}{g^2+g'^2}\begin{pmatrix} W_3^\mu & B^\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_{\mu 3} \\ B_\mu \end{pmatrix}, \tag{4.25}$$

which can be diagonalized by an orthogonal transformation of the form:

$$\begin{pmatrix} W_{\mu 3} \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{4.26}$$

where $c_W \equiv \cos\theta_W$, $s_W \equiv \sin\theta_W$, and θ_W is the weak (or Weinberg) angle. The weak angle can be determined explicitly as follows:

$$\begin{aligned}
&\left(\frac{gW_3^\mu - g'B^\mu}{\sqrt{g^2+g'^2}}\right)^2 \\
&= \frac{1}{g^2+g'^2}\begin{pmatrix} Z^\mu & A^\mu \end{pmatrix}\begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\
&= \frac{1}{g^2+g'^2}\begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \\
&\begin{pmatrix} (gc_W + g's_W)^2 & -gg'\cos 2\theta_W + \frac{(g^2-g'^2)}{2}\sin 2\theta_W \\ -gg'\cos 2\theta_W + \frac{(g^2-g'^2)}{2}\sin 2\theta_W & (gs_W - g'c_W)^2 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}.
\end{aligned} \tag{4.27}$$

From the diagonal condition, we obtain:

$$\tan 2\theta_W = \frac{2gg'}{g^2 - g'^2}. \tag{4.28}$$

By identifying A^μ with the massless photon, we find:

$$\tan\theta_W \equiv t_W = \frac{g'}{g}. \tag{4.29}$$

The solution to both equations is

$$s_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{4.30}$$

In terms of the physical gauge bosons, we obtain

$$(D^\mu H)^\dagger \left(D_\mu H\right) + (D^\mu \phi)^\dagger \left(D_\mu \phi\right) = \frac{g^2}{4} v^2 W^{\mu +} W^-_\mu + \frac{\left(g^2 + g'^2\right)}{8} v^2 Z^\mu Z_\mu + \frac{9}{2} g_L^2 w^2 Z_L^\mu Z_{L\mu}. \tag{4.31}$$

We can identify from the previous expression the mass terms:

$$M_W^2 = \frac{g^2 v^2}{4}, \quad M_Z^2 = \frac{\left(g^2 + g'^2\right) v^2}{4}, \quad M_{Z_L}^2 = 9 g_L^2 w^2 \tag{4.32}$$

4.1.3 Scotogenic neutrino masses

The Yukawa Lagrangian is

$$\begin{aligned} -L_Y =& y^e \overline{L}_L H e_R + y^u \overline{Q}_L \widetilde{H} u_R + y^d \overline{Q}_L H d_R + y^\nu \overline{L}_L S_R \widetilde{\eta} + h \overline{S}_L \nu_R \sigma + M_S^D \overline{S}_L S_R \\ &+ y_1^{e'} \overline{e}''_L H^\dagger L''_R + y_2^{e'} \overline{L}'_L H e'_R + y_3^{e'} \overline{e}''_L \phi e'_R + y_4^{e'} \overline{L}'_L \phi^* L''_R \\ &+ y_1^n \overline{n_L} \widetilde{H}^\dagger L''_R + y_2^n \overline{L}'_L \widetilde{H} n_R + y_3^n \overline{n}_L \phi n_R + \text{h.c.}, \end{aligned} \tag{4.33}$$

where $\widetilde{H} = i\tau^2 H^*$. We also need to note that $h_{k3} = 0$ since only the terms ν_{aR} participate in the neutrino mass generation mechanism, due to the assigned chiral charges of the right handed neutrinos $(4, 4, -5)$. Therefore, this mechanism gives mass to two of the SM neutrinos. The ν_{3R} remains massless and decouples from the rest of the model. Additional terms could be included in the Lagrangian depending on the value of l, but we leave this value unspecified to restrict the analysis to the minimal number of terms. If we were to choose specific values for l, we must do the assignments in such a way that we do not allow tree-level couplings of the form $n_L \left(\nu_L H^0 - e_L H^+\right)$ and similar. The selected $\mathrm{U}(1)_L$ charge for *phi* also avoids that a heavy stable lepton with unacceptably large couplings to Z or H appears.

In addition, one can note that in the limit that the Yukawa couplings $y_3^{e'}$,$y_4^{e'}$,$y_3^n \to 0$, one recovers the global symmetries which in this case separately preserves L, l and $l-3$. As a result, it results natural that these couplings satisfy $y_i \ll 1$, implying that vector-like masses for the new leptons much smaller than w are natural. Furthermore, the small values of the rest of the new lepton couplings $y_i^{e'}$, y_i^n are also natural.

Neutrino masses are not generated at the tree level, they arise as a calculable one-loop contribution via the diagram in Fig. 4.1.

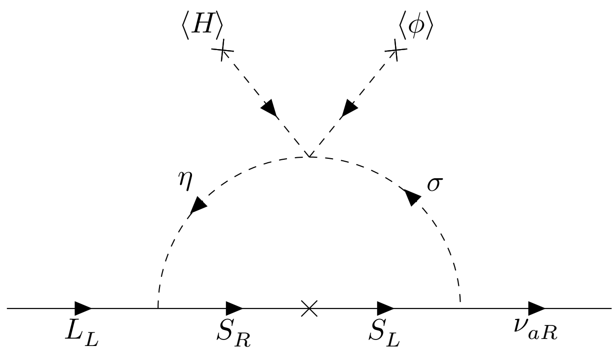


Figure 4.1: One-loop diagram for neutrino masses.

The expression for the neutrino mass matrix in the scotogenic model can be formulated in terms of the one-loop Passarino-Veltman function B_0[105–107]:

$$(m_\nu)_{ij} = \frac{\sin 2\theta}{32\pi^2}\sum_{k,a} y^\nu_{ik} h_{kj} m_{S_k} (-1)^a B_0\left(0, m_a^2, m_{S_k}^2\right), \tag{4.34}$$

Here, m_{S_k} represents the eigenvalues of the Dirac mass matrix M_S^D, m_i (for $i = \varphi_1, \varphi_2$) are the previously found eigenvalues $m_{\varphi^0_{1,2}}$ of the mass eigenbasis of the rotated fields η^0 and σ, $\sin 2\theta$ corresponds to their mixing angle given in Eq. 4.15, and B_0 is defined as:

$$B_0\left(0, m_a^2, m_{S_k}^2\right) = \Delta_\varepsilon + 1 - \frac{m_a^2 \log m_a^2 - m_{S_k}^2 \log m_{S_k}^2}{m_a^2 - m_{S_k}^2}, \tag{4.35}$$

where Δ_ε diverges in the limit $\varepsilon \to 0$. Expanding the previous result, it takes the form:

$$(m_\nu)_{ij} = \frac{\sin 2\theta}{32\pi^2}\sum_{k} y^\nu_{ik} h_{kj} m_{S_k} \left[\frac{m_{\varphi_1}^2}{m_{\varphi_1}^2 - m_{S_k}^2}\ln\frac{m_{\varphi_1}^2}{m_{S_k}^2} - \frac{m_{\varphi_2}^2}{m_{\varphi_2}^2 - m_{S_k}^2}\ln\frac{m_{\varphi_2}^2}{m_{S_k}^2}\right]. \tag{4.36}$$

Concerning dark matter stability in this particular model, we observe that the lightest particle inside the loop is stable. This holds true for both the cases the fermionic and scalar dark matter candidates. All the particles in the internal loop are odd under the remnant $\mathbb{Z}_6$, while the SM particles are even. This implies that any combination of SM fields will be even under the residual subgroup, forbidding all effective operators leading to dark matter decay and preventing mixing with the SM leptons.

4.1.4 Comments and ongoing work

In the ongoing development of this thesis, we are concurrently working on a companion paper that will extend certain aspects covered in this thesis, with a primary focus on the phenomenological facets of our theoretical framework. In this section, we provide a comprehensive overview of the essential theoretical foundations necessary for the ongoing study. Furthermore, we engage in a discourse on the anticipated outcomes and implications within the framework of our model.

In the preceding section, we discussed the possibility of the dark matter candidate being either scalar or fermionic. In the case of a scalar dark matter candidate, the stability criterion designates the lightest among the scalars φ_1^0 and φ_2^0 as stable, assuming the role of dark matter. Consistency with direct detection experiments necessitates a very small coupling between the complex dark matter candidate and the Z-boson. This can be achieved through minimal mixing between φ_1^0 and φ_2^0, where the lightest state is φ_2^0, predominantly representing the scalar singlet σ.

While an in-depth examination of the properties of dark matter candidates is beyond the scope of this work, it will be thoroughly explored in the forthcoming paper. In that paper, we will demonstrate how one can satisfy direct detection constraints and achieve the required relic density. The dark matter candidates may exhibit couplings to the new gauge boson Z_L and the new scalars, potentially yielding the right annihilation cross sections near resonance. There is enough freedom to possibly satisfy the the direct detection constraints coming from experiments.

Moreover, this model encompasses various components that may contribute to explaining the baryon asymmetry of the universe. Our current construction inherently includes new massive states and interactions with CP-violating phases. Consequently, it is intriguing to explore whether this model can effectively explain both the baryon asymmetry and dark matter. The WIMP in this model is a Dirac fermion, offering the potential to realize a theory involving asymmetric dark matter.

If DM consists of WIMPs, it could be the case that DM particles are produced at the LHC and subsequently escape the detectors. For this reason, there are several collaborations dedicated to the search for events where large missing energy is the dominant discriminating signature of DM. In addition, the DM abundance in our universe is likely to be fixed by the thermal freeze-out phenomenon: DM particles, initially present in our universe in thermal equilibrium abundance, annihilate with one another until chemical equilibrium is lost due to the expansion of the universe. The present-day relic density of these particles is predictable and, in the simple case of s-wave self-annihilation of DM in the early universe, it comes out to be

$$\Omega_{\mathrm{DM}} h^2 \simeq \frac{2 \times 2.4 \times 10^{-10} \mathrm{GeV}^{-2}}{\langle \sigma v \rangle_{\mathrm{ann}}}, \tag{4.37}$$

where $\langle \sigma v \rangle_{\mathrm{ann}}$ is the total thermally-averaged annihilation cross section, and the factor of 2 in the numerator is made explicit to emphasize the fact that we are assuming a non-self-conjugate DM particle. This abundance must match the one measured by the Planck collaboration $\Omega^{\text{obs}}_{\text{DM}}h^2 = 0.1199 \pm 0.0027$.

We do not have enough information about the detailed nature of cold dark matter (CDM) except for its relic density. There are major experimental efforts for direct and indirect detection of dark matter particles besides the gravitational effect that it has on the Universe because they must have some connection to the SM particles. Direct detection probes the scattering of dark matter off nuclei in the dark matter detectors, while indirect detection investigates the SM final states from the annihilation of dark matter by cosmic ray detectors.

Curves corresponding to the correct relic abundance serve as benchmark EFT constraints. Part of the ongoing paper includes an analysis of the annihilation channels and the computation of the relic abundance for DM candidates to establish allowed mass ranges. To provide a solid foundation for these calculations, we begin with some general considerations about DM abundance, working within a set of assumptions.

Relic density constraints on thermal DM are often considered non-robust. For a given parameter set, the relic density can vary depending on the inclusion of additional annihilation channels, such as those involving leptons. Conversely, the true relic density might be larger if the dark sector includes various types of DM. Despite these challenges, under a modest set of assumptions, relic density constraints can become substantially more powerful. For instance, in [108], the assumptions include: the DM candidate constitutes 100% of the DM in the universe, the DM annihilation rate is related to the observed density today through the standard thermal production mechanism, the dominant annihilation channel is to Standard Model (SM) fermions via one dark mediator, and the DM couples to u, d quarks, with the coupling to the first generation of quarks no less than the coupling to other SM fermions. Under these assumptions, the relic density constraint provides a range within which the dark sector parameters should lie. The computation of relic density is commonly performed using the software micrOMEGAs[109].

Irrespective of the Dirac or Majorana nature of neutrinos, if $\mathrm{U}(1)_L$ breaks to an even residual $\mathbb{Z}_2 n$ symmetry, there is an associated $0\nu 2n\beta$ decay allowed by the residual symmetry. If neutrinos are Dirac particles, the lowest process allowed by $\mathbb{Z}_{2n}$ symmetry is $0\nu 2n\beta$ decays, with all other lower-dimensional processes being forbidden by the residual $\mathbb{Z}_{2n}$ symmetry. Regarding our model, in the exceptional hypothetical case that a $0\nu 6\beta$ decay was observed in future experiments without positive signals of $0\nu 2k\beta$ decay, with $k < 3$, then neutrinos should be Dirac particles. The same reasoning can be generalized to higher $\mathbb{Z}_m$ symmetries and $0\nu 2n\beta$ decays. The notion that neutrinos are Dirac particles remains viable in the absence of incontrovertible experimental proof of the existence of neutrinoless double-beta decay.

5

Summary and Conclusion

The minimal Standard Model stands as a remarkable achievement in particle physics, providing accurate explanations for numerous experimental observations. However, inherent limitations and persisting questions underscore its incomplete nature. The groundbreaking discovery of neutrino oscillations unveiled the non-negligible masses of neutrinos, challenging the massless assumption of the Standard Model. Additionally, the model falls short in elucidating the mysteries of dark matter, the baryon asymmetry of the universe, and other unresolved phenomena.

Addressing some of these fundamental challenges within the Standard Model, this thesis has crafted an extension based on the scotogenic model, aiming to address neutrino mass generation and introduce viable dark matter candidates. The exploration began with an in-depth examination of neutrino mass generation mechanisms, encompassing the dimension-5 Weinberg operator and the seesaw mechanisms, capable of generating the Weinberg operator at tree level.

Subsequently, we introduced a minimal extension known as the scotogenic model, where neutrino masses arise through a one-loop radiative mechanism, accompanied by the inclusion of dark matter candidates. The versatility of the scotogenic model has led to numerous extensions, accommodating both Dirac and Majorana neutrinos, and incorporating various modifications such as altered particle content and the introduction of new symmetries.

Building upon these extensions, our theoretical framework was presented, featuring the gauging of lepton number U_L, spontaneously broken by a scalar singlet ϕ acquiring a vacuum expectation value. This breaking results in a residual discrete $\mathbb{Z}_6$ symmetry that ensures dark matter stability. The inclusion of this symmetry necessitated the introduction of new particles in the scalar and fermionic sectors, their charges under U_L, and careful anomaly cancellation to maintain theoretical consistency. Our model intriguingly ties dark matter stability to the smallness of neutrino masses.

A dark sector emerges, acting as mediators in the radiative mechanism responsible for neutrino mass generation at the one-loop level. The specific choice of L charges for right-handed neutrinos dictates that only two of the three SM neutrinos acquire mass through this mechanism, leaving the third right-handed neutrino massless and decoupled from the rest of the model.

An additional consequence is the appearance of a new gauge boson Z_L, for which we discussed interactions and computed the mass term. The most plausible dark matter candidate is the lightest of the internal loop particles, odd under the $\mathbb{Z}_6$ symmetry, with potential scalar or fermionic nature. Our model predicts Dirac neutrinos and upholds lepton number conservation.

Despite years of investigation, the true nature and mass generation mechanism of neutrinos remain elusive. Neutrinos might be either Majorana or Dirac particles, and their character may be revealed through observations of lepton number-violating processes such as neutrinoless double-beta decay.

Emphasizing the importance of exploring diverse neutrino mass mechanisms and dark matter candidates, even in marginally viable models, this thesis contributes to the ongoing pursuit of answers. While uncertainties persist, we expect from the acquisition of new data to exclude certain models, gradually narrowing down the viable theories explaining neutrino masses and the nature of dark matter. Until these mysteries are unraveled, it remains uncertain whether dark matter's nature is intricately linked to the smallness of neutrino masses. We anticipate that as answers unfold, they may lead to new questions and avenues for exploration.

Bibliography

[1] Paul Langacker. *The Standard Model and Beyond*. 2nd ed. 2017.

[2] Mathew D. Schwartz. *Quantum Field Theory and the Standard Model*. Cambridge University Press, 2013.

[3] Mark Allen. Srednicki. *Quantum field theory*. Cambridge University Press, 2012, p. 641. ISBN: 9780521864497.

[4] M. H. Ahn et al. "Measurement of neutrino oscillation by the K2K experiment". In: *Phys. Rev. D* 74 (7 2006), p. 072003. DOI: 10.1103/PhysRevD.74.072003. URL: https://link.aps.org/doi/10.1103/PhysRevD.74.072003.

[5] Y. Fukuda et al. "Evidence for Oscillation of Atmospheric Neutrinos". In: *Physical Review Letters* 81 (8 Aug. 1998), p. 1562. ISSN: 10797114. DOI: 10.1103/PhysRevLett.81.1562. URL: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.81.1562.

[6] A. Gando et al. "Constraints on θ_{13} from a three-flavor oscillation analysis of reactor antineutrinos at KamLAND". In: *Phys. Rev. D* 83 (5 2011), p. 052002. DOI: 10.1103/PhysRevD.83.052002.

[7] P. A.R. Ade et al. "Planck 2015 results - XIII. Cosmological parameters". In: *Astronomy and Astrophysics* 594 (Oct. 2016), A13. ISSN: 0004-6361. DOI: 10.1051/0004-6361/201525830.

[8] P. Salucci and A. Borriello. *The Intriguing Distribution of Dark Matter in Galaxies*. 2002. arXiv: astro-ph/0203457 [astro-ph].

[9] Bing Lin Young. *A survey of dark matter and related topics in cosmology*. Apr. 2017. DOI: 10.1007/s11467-016-0583-4.

[10] W. Pauli. "Dear radioactive ladies and gentlemen". In: *Phys. Today* 31N9 (1978), p. 27.

[11] Wolfgang Pauli. “Aufsätze und Vorträge über Physik und Erkenntnistheorie”. In: *Aufsätze und Vorträge über Physik und Erkenntnistheorie* (1961). DOI: 10.1007/978-3-663-07092-4.

[12] E. Fermi. “Versuch einer Theorie der β-Strahlen. I”. In: *Zeitschrift für Physik* 88 (3-4 Mar. 1934), pp. 161–177. ISSN: 14346001. DOI: 10.1007/BF01351864/METRICS.

[13] Francis Perrin. “Possibilité d’émission de particules neutres de masse intrinsèque nulle dans les radioactivités β”. In: *Comptes rendus* 197 (1933), pp. 1625–1627.

[14] H. Bethe and R. Peierls. “The “Neutrino””. In: *Nature 1934 133:3362* 133 (3362 1934), pp. 532–532. ISSN: 1476-4687. DOI: 10.1038/133532a0. URL: https://www.nature.com/articles/133532a0.

[15] Q. R. Ahmad et al. “Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory”. In: *Physical Review Letters* 89 (1 June 2002), p. 011301. ISSN: 10797114. DOI: 10.1103/PhysRevLett.89.011301. URL: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.89.011301.

[16] A. Aguilar et al. “Evidence for neutrino oscillations from the observation ofElectron Anti-neutrinos in a Muon Anti-Neutrino Beam”. In: *Physical Review D* 64.11 (2001). DOI: 10.1103/physrevd.64.112007. URL: https://doi.org/10.1103%2Fphysrevd.64.112007.

[17] Y. Fukuda et al. “Neutrino-induced upward stopping muons in Super-Kamiokande”. In: *Physics Letters B* 467.3-4 (1999), pp. 185–193. DOI: 10.1016/s0370-2693(99)01188-0. URL: https://doi.org/10.1016%2Fs0370-2693%2899%2901188-0.

[18] Kun Feng Lyu et al. “Self-interacting neutrinos: Solution to Hubble tension versus experimental constraints”. In: *Physical Review D* 103 (1 Jan. 2021), p. 015004. ISSN: 24700029. DOI: 10.1103/PHYSREVD.103.015004/FIGURES/2/MEDIUM. URL: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.103.015004.

[19] Scott Dodelson and Lawrence M. Widrow. “Sterile neutrinos as dark matter”. In: *Physical Review Letters* 72.1 (1994), pp. 17–20. DOI: 10.1103/physrevlett.72.17. URL: https://doi.org/10.1103%2Fphysrevlett.72.17.

[20] Debasish Majumdar. *Dark matter: an introduction*. CRC Press, 2014.

[21] Ernest Ma. "Verifiable radiative seesaw mechanism of neutrino mass and dark matter". In: *Phys. Rev. D* 73 (7 2006), p. 077301. DOI: 10.1103/PhysRevD.73.077301. URL: https://link.aps.org/doi/10.1103/PhysRevD.73.077301.

[22] Yasaman Farzan and Ernest Ma. "Dirac neutrino mass generation from dark matter". In: *Phys. Rev. D* 86 (3 2012), p. 033007. DOI: 10.1103/PhysRevD.86.033007. URL: https://link.aps.org/doi/10.1103/PhysRevD.86.033007.

[23] M. Hirsch et al. "WIMP dark matter as radiative neutrino mass messenger". In: *Journal of High Energy Physics* 2013.10 (2013). DOI: 10.1007/jhep10(2013)149. URL: https://doi.org/10.1007%2Fjhep10%282013%29149.

[24] Alexander Merle et al. "Consistency of WIMP Dark Matter as radiative neutrino mass messenger". In: *Journal of High Energy Physics* 2016.7 (2016), pp. 1–17.

[25] M. Hirsch et al. "Discrete dark matter". In: *Physical Review D* 82.11 (2010). DOI: 10.1103/physrevd.82.116003. URL: https://doi.org/10.1103%2Fphysrevd.82.116003.

[26] M. S. Boucenna et al. "Phenomenology of dark matter from A 4 flavor symmetry". In: *Journal of High Energy Physics* 2011.5 (2011). DOI: 10.1007/jhep05(2011)037. URL: https://doi.org/10.1007%2Fjhep05%282011%29037.

[27] Ernest Ma. *Verifiable Radiative Seesaw Mechanism of Neutrino Mass and Dark Matter.* 2006.

[28] A. Zee. *Group Theory in a Nutshell for Physicists.* 2016.

[29] Esko Keski-Vakkuri et al. *Mathematical Methods for Physics. An Introduction to Group Theory, Topology and Geometry.* Cambridge University Press, 2022.

[30] Brian Hall. *Lie Groups, Lie Algebras, and Representations - An Elementary Introduction.* 2nd. Springer, 2015. URL: http://www.springer.com/series/136.

[31] J. M. Butterworth. "The Standard Model: How far can it go and how can we tell?" In: *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 374 (2075 Aug. 2016). ISSN: 1364503X. DOI: 10.1098/RSTA.2015.0260. URL: http://dx.doi.org/10.1098/rsta.2015.0260.

[32] T. E. Chupp et al. “Electric dipole moments of atoms, molecules, nuclei, and particles”. In: *Reviews of Modern Physics* 91.1 (2019). DOI: 10.1103/revmodphys.91.015001. URL: https://doi.org/10.1103%2Frevmodphys.91.015001.

[33] Particle Data Group et al. “Review of Particle Physics”. In: *Progress of Theoretical and Experimental Physics* 2022 (8 Aug. 2022), pp. 1–878. ISSN: 14346044. DOI: 10.1093/PTEP/PTAC097. URL: https://dx.doi.org/10.1093/ptep/ptac097.

[34] P. W. Higgs. “Broken symmetries, massless particles and gauge fields”. In: *Phys.Lett.* 12 (2 Sept. 1964), pp. 132–133. ISSN: 00319163. DOI: 10.1016/0031-9163(64)91136-9.

[35] G. Arnison et al. “Experimental Observation of Isolated Large Transverse Energy Electrons with Associated Missing Energy at $\sqrt{\{s\}} = 540$ GeV”. In: *Phys.Lett.B* 122 (1 Feb. 1983), pp. 103–116. ISSN: 03702693. DOI: 10.1016/0370-2693(83)91177-2.

[36] G. Arnison et al. “Experimental Observation of Lepton Pairs of Invariant Mass Around 95-GeV/c**2 at the CERN SPS Collider”. In: *Phys.Lett.B* 126 (5 July 1983), pp. 398–410. ISSN: 03702693. DOI: 10.1016/0370-2693(83)90188-0.

[37] Georges Aad et al. “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC”. In: *Phys. Lett. B* 716 (2012), pp. 1–29. DOI: 10.1016/j.physletb.2012.08.020. arXiv: 1207.7214 [hep-ex].

[38] S. L. Glashow et al. “Weak Interactions with Lepton-Hadron Symmetry”. In: *Physical Review D* 2 (7 Oct. 1970), p. 1285. ISSN: 05562821. DOI: 10.1103/PhysRevD.2.1285. URL: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.2.1285.

[39] Nicola Cabibbo. “Unitary Symmetry and Leptonic Decays”. In: *Physical Review Letters* 10 (12 June 1963), p. 531. ISSN: 00319007. DOI: 10.1103/PhysRevLett.10.531. URL: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.10.531.

[40] Makoto Kobayashi and Toshihide Maskawa. “CP-Violation in the Renormalizable Theory of Weak Interaction”. In: *Progress of Theoretical Physics* 49 (2 Feb. 1973), pp. 652–657. ISSN: 0033-068X. DOI: 10.1143/PTP.49.652. URL: https://dx.doi.org/10.1143/PTP.49.652.

[41] Karl Kåre Fridell. “Phenomenology of Baryogenesis and Neutrino Physics: From Effective Field Theory to Simplified Models”. In: (2022).

[42] Debasish Majumdar. *Dark Matter: An Introduction*. Taylor & Francis Group, 2015.

[43] Zwicky F. "Die Rotverschiebung von extragalaktischen Nebeln". In: *AcHPh* 6 (1933), pp. 110–127. ISSN: 0018-0238.

[44] N. Aghanim et al. "Planck2018 results: VI. Cosmological parameters". In: *Astronomy and Astrophysics* 641 (2020), A6. ISSN: 1432-0746. DOI: 10.1051/0004-6361/201833910.

[45] M. Kowalski et al. "Improved Cosmological Constraints from New, Old, and Combined Supernova Data Sets". In: *The Astrophysical Journal* 686.2 (2008), pp. 749–778. ISSN: 1538-4357. DOI: 10.1086/589937.

[46] Marco Taoso et al. "Dark matter candidates: a ten-point test". In: *Journal of Cosmology and Astroparticle Physics* 2008.03 (2008). ISSN: 1475-7516. DOI: 10.1088/1475-7516/2008/03/022.

[47] Gianfranco Bertone. "Particle dark matter: Observations, models and searches". In: (Jan. 2010), pp. 1–738. DOI: 10.1017/CBO9780511770739.

[48] Benjamin W. Lee and Steven Weinberg. "Cosmological Lower Bound on Heavy-Neutrino Masses". In: *Phys. Rev. Lett.* 39 (4 1977), pp. 165–168. DOI: 10.1103/PhysRevLett.39.165.

[49] Gerard Jungman et al. "Supersymmetric dark matter". In: *Physics Reports* 267.5-6 (1996), pp. 195–373. ISSN: 0370-1573. DOI: 10.1016/0370-1573(95)00058-5.

[50] Gintaras Duda and Katherine Garrett. "Dark matter: A primer". In: *Advances in Astronomy* 2011 (2011). ISSN: 16877969. DOI: 10.1155/2011/968283.

[51] C. L. Cowan et al. "Detection of the Free Neutrino: a Confirmation". In: *Science* 124.3212 (1956), pp. 103–104. DOI: 10.1126/science.124.3212.103. eprint: https://www.science.org/doi/pdf/10.1126/science.124.3212.103. URL: https://www.science.org/doi/abs/10.1126/science.124.3212.103.

[52] Samoil Bilenky. *Introduction to the Physics of Massive and Mixed Neutrinos*. 2nd ed. Springer, 2018. DOI: https://doi.org/10.1007/978-3-319-74802-3.

[53] B. Pontecorvo. "Inverse beta processes and nonconservation of lepton charge". In: *Zh. Eksp. Teor. Fiz.* 34 (1957), p. 247.

[54] B. Pontecorvo. "Mesonium and anti-mesonium". In: *Sov. Phys. JETP* 6 (1957), p. 429.

[55] G. Danby et al. “Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos”. In: *Phys. Rev. Lett.* 9 (1 1962), pp. 36–44. DOI: 10.1103/PhysRevLett.9.36. URL: https://link.aps.org/doi/10.1103/PhysRevLett.9.36.

[56] Ziro Maki et al. “Remarks on the unified model of elementary particles”. In: *Prog. Theor. Phys.* 28 (1962), pp. 870–880. DOI: 10.1143/PTP.28.870.

[57] B. Pontecorvo. “Neutrino Experiments and the Problem of Conservation of Leptonic Charge”. In: *Zh. Eksp. Teor. Fiz.* 53 (1967), pp. 1717–1725.

[58] Raymond Davis et al. “Search for Neutrinos from the Sun”. In: *Phys. Rev. Lett.* 20 (21 1968), pp. 1205–1209. DOI: 10.1103/PhysRevLett.20.1205. URL: https://link.aps.org/doi/10.1103/PhysRevLett.20.1205.

[59] K. Kodama et al. “Observation of tau neutrino interactions”. In: *Physics Letters B* 504.3 (2001), pp. 218–224. DOI: 10.1016/s0370-2693(01)00307-0. URL: https://doi.org/10.1016%2Fs0370-2693%2801%2900307-0.

[60] S.M. Bilenky et al. “Phenomenology of neutrino oscillations”. In: *Progress in Particle and Nuclear Physics* 43 (1999), pp. 1–86. DOI: 10.1016/s0146-6410(99)00092-7.

[61] J. D. Vergados et al. “Theory of neutrinoless double beta decay”. In: *Reports on Progress in Physics* 75 (10 May 2012). DOI: 10.1088/0034-4885/75/10/106301. URL: http://arxiv.org/abs/1205.0649%20http://dx.doi.org/10.1088/0034-4885/75/10/106301.

[62] J. Schechter and J. W.F. Valle. “Neutrinoless Double beta Decay in SU(2) x U(1) Theories”. In: *Phys.Rev.D* 25 (11 1982), p. 2951. ISSN: 05562821. DOI: 10.1103/PHYSREVD.25.2951.

[63] J. Aalbers et al. *A next-generation liquid xenon observatory for dark matter and neutrino physics*. Jan. 2023. DOI: 10.1088/1361-6471/ac841a.

[64] Samoil M. Bilenky et al. “On Oscillations of Neutrinos with Dirac and Majorana Masses”. In: *Phys. Lett. B* 94 (1980), pp. 495–498. DOI: 10.1016/0370-2693(80)90927-2.

[65] Steven Weinberg. “Baryon and Lepton Nonconserving Processes”. In: *Phys. Rev. Lett.* 43 (1979), pp. 1566–1570. DOI: 10.1103/PhysRevLett.43.1566.

[66] Yi Cai et al. “From the trees to the forest: a review of radiative neutrino mass models”. In: (June 2017). DOI: 10.3389/fphy.2017.00063. URL: http://arxiv.org/abs/1706.08524%20http://dx.doi.org/10.3389/fphy.2017.00063.

[67] Georg G. Raffelt. “Particle physics from stars”. In: *Ann. Rev. Nucl. Part. Sci.* 49 (1999), pp. 163–216. DOI: 10.1146/annurev.nucl.49.1.163. arXiv: hep-ph/9903472.

[68] Subir Sarkar. “Big bang nucleosynthesis and physics beyond the standard model”. In: *Reports on Progress in Physics* 59.12 (1996), pp. 1493–1609. DOI: 10.1088/0034-4885/59/12/001.

[69] E. Kh. Akhmedov. *Neutrino physics*. 2000. arXiv: hep-ph/0001264 [hep-ph].

[70] Peter Minkowski. “$\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?” In: *Phys. Lett. B* 67 (1977), pp. 421–428. DOI: 10.1016/0370-2693(77)90435-X.

[71] K. Scholberg. *Atmospheric Neutrinos at Super-Kamiokande*. 1999. arXiv: hep-ex/9905016 [hep-ex].

[72] R. Foot et al. “See-saw neutrino masses induced by a triplet of leptons”. In: *Zeitschrift für Physik C Particles and Fields* 44 (3 Sept. 1989), pp. 441–444. ISSN: 01709739. DOI: 10.1007/BF01415558/METRICS. URL: https://link.springer.com/article/10.1007/BF01415558.

[73] J. Schechter and J. W. F. Valle. “Neutrino masses in SU(2) $\bigotimes$ U(1) theories”. In: *Phys. Rev. D* 22 (9 1980), pp. 2227–2235. DOI: 10.1103/PhysRevD.22.2227.

[74] M. Magg and Ch Wetterich. “Neutrino mass problem and gauge hierarchy”. In: *Physics Letters B* 94 (1 July 1980), pp. 61–64. ISSN: 0370-2693. DOI: 10.1016/0370-2693(80)90825-4.

[75] Alexander Natale. “Radiative Neutrino Mass, Dark Matter, Flavor Symmetry, and Collider Signatures”. In: (2015). URL: https://escholarship.org/uc/item/0710h931.

[76] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to quantum field theory*. Reading, USA: Addison-Wesley, 1995. ISBN: 978-0-201-50397-5.

[77] Florian Bonnet et al. “Systematic study of the d = 5 Weinberg operator at one-loop order”. In: *Journal of High Energy Physics* 2012.7 (2012). DOI: 10.1007/jhep07(2012)153.

[78] ERNEST MA and DAIJIRO SUEMATSU. “FERMION TRIPLET DARK MATTER AND RADIATIVE NEUTRINO MASS”. In: *Modern Physics Letters A* 24.08 (2009), pp. 583–589. ISSN: 1793-6632. DOI: 10.1142/s021773230903059x.

[79] Yasaman Farzan. “Minimal model linking two great mysteries: Neutrino mass and dark matter”. In: *Physical Review D* 80.7 (2009). ISSN: 1550-2368. DOI: 10.1103/physrevd.80.073009.

[80] Dieter Hehn and Alejandro Ibarra. “A radiative model with a naturally mild neutrino mass hierarchy”. In: *Physics Letters B* 718.3 (2013), pp. 988–991. ISSN: 0370-2693. DOI: 10.1016/j.physletb.2012.11.034.

[81] Wen-Bin Lu and Pei-Hong Gu. “Mixed Inert scalar triplet dark matter, radiative neutrino masses and leptogenesis”. In: *Nuclear Physics B* 924 (2017), pp. 279–311. ISSN: 0550-3213. DOI: 10.1016/j.nuclphysb.2017.09.005.

[82] Yi Cai et al. “RνMDM and lepton flavor violation”. In: *Journal of High Energy Physics* 2011 (12 Dec. 2011), pp. 1–22. ISSN: 11266708. DOI: 10.1007/JHEP12(2011)054/METRICS.

[83] Chuan-Hung Chen et al. “Neutrino masses, leptogenesis and decaying dark matter”. In: *Journal of Cosmology and Astroparticle Physics* 2009.10 (2009), pp. 001–001. ISSN: 1475-7516. DOI: 10.1088/1475-7516/2009/10/001.

[84] Romy H. S. Budhi et al. “Inflation due to a nonminimal coupling of singlet scalars in the radiative seesaw model”. In: *Physical Review D* 93.1 (2016). ISSN: 2470-0029. DOI: 10.1103/physrevd.93.013022.

[85] Ernest Ma. *Supersymmetric Model of Radiative Seesaw Majorana Neutrino Masses.* 2006. arXiv: hep-ph/0607142 [hep-ph].

[86] T. de Boer et al. “Anomaly-free dark matter models with one-loop neutrino masses and a gauged U(1) symmetry”. In: (Sept. 2023). URL: http://arxiv.org/abs/2309.06920.

[87] Ya. B. Zeldovich et al. “Cosmological Consequences of the Spontaneous Breakdown of Discrete Symmetry”. In: *Zh. Eksp. Teor. Fiz.* 67 (1974), pp. 3–11.

[88] Jonathan L. Feng and Jason Kumar. “Dark-Matter Particles without Weak-Scale Masses or Weak Interactions”. In: *Physical Review Letters* 101.23 (2008). DOI: 10.1103/physrevlett.101.231301. URL: https://doi.org/10.1103%2Fphysrevlett.101.231301.

[89] Ernest Ma et al. "SCOTOGENIC A4 NEUTRINO MODEL FOR NONZERO θ13 AND LARGE δCP". In: *International Journal of Modern Physics A* 27 (2012), p. 1250134. URL: https://api.semanticscholar.org/CorpusID:118532842.

[90] Julio Leite et al. "Scotogenic dark matter and Dirac neutrinos from unbroken gauged $B - L$ symmetry". In: (Mar. 2020). DOI: 10.1016/j.physletb.2020.135537. URL: http://arxiv.org/abs/2003.02950%20http://dx.doi.org/10.1016/j.physletb.2020.135537.

[91] Ernest Ma and Alexander Natale. "Scotogenic Z_2 or $U(1)_D$ Model of Neutrino Mass with $\Delta(27)$ Symmetry". In: *Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics* 734 (Mar. 2014), pp. 403–405. DOI: 10.1016/j.physletb.2014.05.070. URL: http://arxiv.org/abs/1403.6772%20http://dx.doi.org/10.1016/j.physletb.2014.05.070.

[92] Ernest Ma. "Derivation of Dark Matter Parity from Lepton Parity". In: *Physical Review Letters* 115.1 (2015). DOI: 10.1103/physrevlett.115.011801. URL: https://doi.org/10.1103%2Fphysrevlett.115.011801.

[93] Ernest Ma and Alexander Natale. "Dark matter with flavor symmetry and its collider signature". In: *Physics Letters B* 740 (2015), pp. 80–82. DOI: 10.1016/j.physletb.2014.11.032. URL: https://doi.org/10.1016%2Fj.physletb.2014.11.032.

[94] Ernest Ma. "Tribimaximal Neutrino Mixing from a Supersymmetric Model with A_4 Family Symmetry". In: *Physical Review D - Particles, Fields, Gravitation and Cosmology* 73 (5 Nov. 2005). DOI: 10.1103/PhysRevD.73.057304. URL: http://arxiv.org/abs/hep-ph/0511133%20http://dx.doi.org/10.1103/PhysRevD.73.057304.

[95] Sean Fraser et al. "Scotogenic inverse seesaw model of neutrino mass". In: *Physics Letters B* 737 (2014), pp. 280–282. DOI: 10.1016/j.physletb.2014.08.069. URL: https://doi.org/10.1016%2Fj.physletb.2014.08.069.

[96] Sean Fraser and Ernest Ma. "Anomalous Higgs Yukawa couplings". In: *Europhysics Letters* 108 (1 Oct. 2014), p. 11002. ISSN: 0295-5075. DOI: 10.1209/0295-5075/108/11002. URL: https://iopscience.iop.org/article/10.1209/0295-5075/108/11002%20https://iopscience.iop.org/article/10.1209/0295-5075/108/11002/meta.

[97] Corey Kownacki et al. "Generalized gauge U(1) family symmetry for quarks and leptons". In: *Physics Letters B* 766 (Mar. 2017), pp. 149–152. ISSN: 0370-2693. DOI: 10.1016/J.PHYSLETB.2017.01.013.

[98] Pavel Fileviez Pérez and Mark B. Wise. "Baryon and lepton number as local gauge symmetries". In: *Physical Review D - Particles, Fields, Gravitation and Cosmology* 82 (1 July 2010). ISSN: 15507998. DOI: 10.1103/PhysRevD.82.011901.

[99] Martin Hirsch et al. "Can one ever prove that neutrinos are Dirac particles?" In: *Physics Letters, Section B: Nuclear, Elementary Particle and High-Energy Physics* 781 (June 2018), pp. 302–305. ISSN: 03702693. DOI: 10.1016/j.physletb.2018.03.073.

[100] Joan Elias-Miró et al. "Stabilization of the electroweak vacuum by a scalar threshold effect". In: *Journal of High Energy Physics* 2012.6 (2012). DOI: 10.1007/jhep06(2012)031. URL: https://doi.org/10.1007%2Fjhep06%282012%29031.

[101] Brian Batell et al. "Singlet assisted vacuum stability and the Higgs to diphoton rate". In: *Journal of High Energy Physics* 2013.1 (2013). DOI: 10.1007/jhep01(2013)135.

[102] Alfredo Aranda et al. "Electroweak phase transition in a model with gauged lepton number". In: *Journal of High Energy Physics* 2015 (1 Jan. 2015). ISSN: 10298479. DOI: 10.1007/JHEP01(2015)070.

[103] Pedro Schwaller et al. "Dark Matter and Vector-like Leptons From Gauged Lepton Number". In: (May 2013). DOI: 10.1103/PhysRevD.88.035001. URL: http://arxiv.org/abs/1305.1108%20http://dx.doi.org/10.1103/PhysRevD.88.035001.

[104] Marcela Carena et al. "Z' gauge bosons at the Fermilab Tevatron". In: *Phys. Rev. D* 70 (2004), p. 093009. DOI: 10.1103/PhysRevD.70.093009.

[105] G Passarino and M Veltman. *ONE-LOOP CORRECTIONS FOR e+e-ANNIHILATION INTO lt+D-IN THE WEINBERG MODEL*. 1979.

[106] Cesar Bonilla et al. "Dark matter stability and Dirac neutrinos using only Standard Model symmetries". In: (Dec. 2018). DOI: 10.1103/PhysRevD.101.033011. URL: http://dx.doi.org/10.1103/PhysRevD.101.033011.

[107] Pablo Escribano et al. “Generalizing the Scotogenic model”. In: (Apr. 2020). DOI: 10.1007/JHEP07(2020)097. URL: http://arxiv.org/abs/2004.05172%20http://dx.doi.org/10.1007/JHEP07(2020)097.

[108] Giorgio Busoni et al. “Making the Most of the Relic Density for Dark Matter Searches at the LHC 14 TeV Run”. In: (Oct. 2014). DOI: 10.1088/1475-7516/2015/03/022.

[109] G. Bélanger et al. “micrOMEGAs_3: A program for calculating dark matter observables”. In: *Computer Physics Communications* 185.3 (2014), pp. 960–985. ISSN: 0010-4655. DOI: 10.1016/j.cpc.2013.10.016.

León, Guanajuato, 9 de febrero de 2024

Dr. David Delepine

Director DCI

Presente

Por este conducto le informo que he leído la tesis titulada *"Massive Neutrinos: A Scotogenic Extension with Gauged Lepton Number"* que para obtener el grado de Maestro en Física ha formulado el L.F. **José Andrés Enríquez López** bajo la dirección del Dr. Carlos Alberto Vaquera Araujo.

En mi opinión, este trabajo reúne las características de calidad y forma para el grado al que se aspira, por lo cual no tengo inconveniente en que se realice la defensa correspondiente.

Sin otro particular, reciba un cordial saludo.

Atentamente

Dr. Mauro Napsuciale Mendivil

Profesor Titular C

Sinodal

c.c.p. archivo

DEPARTAMENTO DE FISICA, DIVISION DE CIENCIAS E INGENIERÍAS, CAMPUS LEÓN

Loma del Bosque 103, Fracc. Lomas del Campestre C.P. 37150 León, Gto., Ap. Postal E-143 C.P. 37000 Tel. (477) 788-5100, Fax: (477) 788-5100 ext. 8410, http://www.fisica.ugto.mx

UNIVERSIDAD DE
GUANAJUATO

León, Guanajuato, 19 de febrero de 2024

Dr. David Delepine
Director
División de Ciencias e Ingenierías
PRESENTE

Por medio de la presente me permito informar que he leído la tesis titulada **“Massive Neutrinos: A Scotogenic Extension with Gauged Lepton Number”**, que para obtener el grado de Maestría en Física ha sido elaborada por el **Lic. José Andrés Enríquez López** bajo mi asesoría. En mi opinión, la tesis cumple con los requisitos de calidad correspondientes al grado académico al que se aspira. Las correcciones sugeridas por mi parte han sido atendidas, por lo cual recomiendo se proceda a la defensa de la tesis.

Sin más por el momento quedo a sus órdenes para cualquier aclaración.

Atentamente

Dr. Carlos Alberto Vaquera Araujo
Investigador por México Conahcyt
Departamento de Física
DCI, Campus León
vaquera@fisica.ugto.mx

UNIVERSIDAD DE GUANAJUATO

León, Guanajuato, 29 de enero de 2024

Dr. David Delepine
Director de la División de Ciencias e Ingenierías
Campus León, Universidad de Guanajuato
PRESENTE

Estimado Dr. Delepine:

Por este medio, me permito informarle que he leído la tesis titulada **"Massive neutrinos: A scotogenic extension with gauged lepton number"** que realizó el Licenciado en Física **José Andrés Enríquez López** como requisito para obtener el grado de Maestro en Física.

Considero que el trabajo de maestría realizado por Andrés es muy completo y que aporta conocimiento de relevancia en el campo de la física de neutrinos. Considero que su trabajo reúne los requisitos necesarios de calidad e interés académico para que sea defendida en un examen de grado, razón por la cual extiendo mi aval para que así se proceda.

Sin más que agregar, agradezco su atención y aprovecho la ocasión para enviarle un cordial saludo.

ATENTAMENTE
"LA VERDAD OS HARÁ LIBRES"

Dr. Juan Barranco Monarca
División de Ciencias e Ingenierías UG

29 de enero de 2024
EMAIL: epeinado@fisica.unam.mx

Dr. David Delepine
Director
División de Ciencias e Ingenierías
Universidad de Guanajuato

Por medio de la presente me permito informar que he revisado el trabajo titulado **"Massive Neutrinos: A Scotogenic Extension with Gauged Lepton Number"** que para obtener el grado de Maestro en Física presenta el estudiante **José Andrés Enríquez López**.

Considero que el trabajo cumple con los requisitos para otorgarle dicho grado, por lo que no tengo inconveniente en que se realicen los procedimientos necesarios para la presentación del examen de grado ante el comité respectivo.

Sin otro particular, aprovecho la ocasión para enviarle un cordial saludo.

Atentamente,

Eduardo Peinado Rodríguez
Investigador Titular A de T. C.
Instituto de Física UNAM