Cosmological solutions for a two branes system in a generalized Randall-Sundrum model

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Abstract

This present thesis is devoted to the study of dynamical and cosmological solutions for a two-branes system, in which the fifth dimension is compactifed on a circle. Under a particular hypothesis, which is used to integrate Einstein's equations, a new set of solutions is derived and they reveals that the cosmological parameters in both branes are related, and this interdependence is due to the bulk geometry. By supposing one of the two branes is dominated, first by a single component and then by a real scalar field, We will analyze what conditions are necessary to obtain inflation in our visible brane. This is achieved by analyzing the corresponding dynamical system emerging from the modified Friedmann equations, establishing what are the limits where these models are valid. A characteristic of these types of models is the ability to explore different types of cosmological solutions according to the shape and components of the universe.

I dedicate my dissertation work to my family and many friends. I want to express a special feeling of gratitude to my beloved wife Claudia Mireya Hernández and my parents, Juan Pérez Escobar and Verónica Pérez Pío, whose words of encouragement and push for tenacity ring in my ears. My brothers René, Uriel and Oscar, although they are distant, have never left my side and are very special.

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I would like to thank all the people who contributed in some way to the work described in this thesis. First and foremost, I thank my academic advisor, Dr. Luis Urena López, for accepting me into his group. During my tenure, he contributed to a rewarding graduate school experience by giving me intellectual freedom in my work, supporting my attendance at various conferences, engaging me in new ideas, and demanding a high quality of work in all my endeavors. Additionally, I would like to thank Professors Ruben Cordero and Miguel Aspeitia who have enriched my thesis work and publications with lengthy discussions and proposals.

Publications

During my time in the doctorate program, six papers were published, two of which were accepted by the General Relativity and Gravitation journal:

- Cosmological constraints for a two brane-world system with single equation of state (2011) [69], https://arxiv.org/abs/1208.4385
- Two-brane system in a vacuum bulk with a single equation of state (2012) [70], https://arxiv.org/abs/1208.5789
- Cosmological constraints in a two branes system for a vacuum bulk (2012) [71] https://arxiv.org/abs/1210.5939
- Cosmological solutions for a two-branes system in a vacuum bulk (2013) [74] https://arxiv.org/abs/1306.2888
- Novel ansatz to obtain inflation in brane-worlds: the dynamical perspective (2014) [72], https://link.springer.com/article/10.1007/s10714-014-1804-1
- Cosmological constraints for a two branes system in a vacuum bulk (2015) [73], https://link.springer.com/article/10.1007/s10714-014-1847-3

Contents

List of Figures

List of Tables

1

Introduction

For years scientists have speculated about how our universe evolved: how it emerged, what is its shape, what are its limits and where it is going. Undoubtedly the most daring questions inquire about its origin and what have exist before its formation, giving explanations that range from the religious to the purely speculative ones [22]. Physics, however, can help us with some clues about how everything could have become visible: with the proposal of some hypothetical scenarios, one can apply the basic laws (Newtonian and Einsteinian) and achieve some important conclusions. It should be clear here that there is no guarantee that the known laws, applicable to the solar system, are also valid at scales as large as clusters of galaxies; however, the hypothesis known as the cosmological principle assures us that the universe is homogeneous and isotropic so that anywhere and in any direction, the laws of nature are exactly the same [46].

Without underestimating the knowledge that was forged throughout history, we can say with certainty that the study of the universe as a whole has its origin in the first half of the twentieth century. In the last century, cosmologists have made great contributions concerning the size of our universe (at least the visible part) and its probable age, begining with the development in 1915 of Albert Einstein's general theory of relativity, followed by major observational discoveries in the 1920s: first, Edwin Hubble discovered that the universe contains a huge number of external galaxies beyond our own Milky Way; then, work by Vesto Slipher and others showed that the universe is expanding. Edwin Hubble, known for his famous law of redshift, established that galaxies are moving away from each other at a speed proportional to their distance, which led him to conjecture that in early times the galaxies were so close together that they could have emerged from a large concentration of energy as a result of a huge explosion (explosion here understood as an expansion of space, time and matter together) Moving on from these discoveries, scientists have made great efforts to explain what exactly happened at the moment of the Big Bang. Recent observations of the cosmic background radiation shed light on the first minutes of the birth of the universe as well as the exact composition of its primary components.

Knowledge of the structure and components of all visible things and the laws that govern them, plays a crucial role in the correct understanding of the different stages in which the universe has been gestated. The most successful theories suggest the idea of an initial stage, where space and time has been "conceived" from a singularity (Big Bang) at scales as large as those hypothesized by Grand Unified Theory (GUT), and that the universe emerged about 13.8 billion years ago with a homogeneous and isotropic distribution of matter at very high temperature and density, which has been expanding and cooling since then[87] followed by a stage of exponential inflation, which cooled the universe to enter in a stage of relaxation and subsequent reheating. Finally the large potential energy of the inflaton field decays into particles and fills the Universe with Standard Model particles, including electromagnetic radiation, starting the radiation dominated phase of the universe [41]. The Lambda cold dark matter (ΛCDM) model is a parametrization of the Big Bang cosmological model in which the universe contains a cosmological constant, denoted by Lambda, associated with dark energy, and cold dark matter. It is frequently referred to as the standard model of Big Bang cosmology, because it is the simplest model that provides a reasonably good account of the majority of the properties of the cosmos. Despite the large observational technological advancement, which has proven to be true at earlier stages, other questions plague cosmologists, especially those having to do with the formation of structure, the mechanism that generates inflation and also stops it, the dark energy and dark matter, the current accelerated expansion, etc. [52]. And it is from these "gaps" in the standard theory of cosmology (and related theories) that many theoretical scientists have been motivated themselves to develop more sophisticated models of the standard ΛCDM, trying to explain natural phenomena that remain mysterious, but others tend to develop completely different

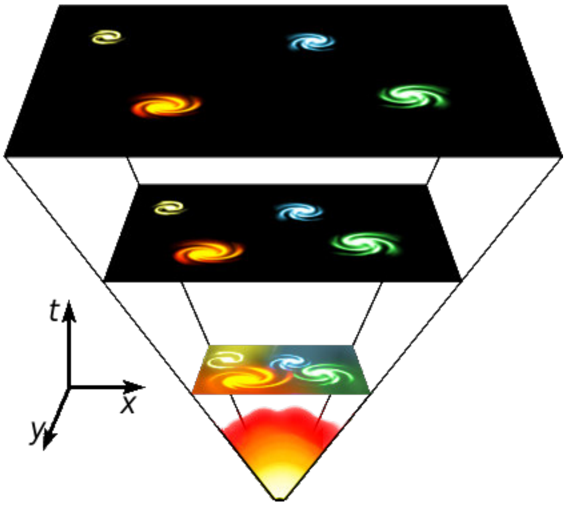

Figure 1.1 According to the Big Bang model, the universe expanded from an extremely dense and hot state and continues to expand today. If the known laws of physics are extrapolated beyond where they are valid, there is a singularity. Modern measurements place this moment at approximately 13.8 billion years ago, which is thus considered the age of the universe. Image taken from `https://en.wikipedia.org/wiki/File:Universe_expansion2.png`

theories [51].

As an alternative solution to the challenges arising from the standard cosmology, in the last century multidimensional models have emerged, which raise the possibility that the universe has more than four dimensions, which can have different sizes, and the 4D cosmology is recovered on a brane[1] [20]. One of the first proposals considered an extra dimension to Einstein's equations; Its effect on the dynamic equations is in the introduction of new energy modes called Kaluza-Klein tower (see [19, 67] for a pedagogical review). One of the difficulties of this model is that it considerably limits the possibility of experimentally detecting these modes of energy; an analysis of results from the LHC in December 2010 severely restrict theories with large extra dimensions [38]. While these models were proposed as an attempt to solve the problem of the hierarchy and the unification of forces, they have also been the basis for tackling issues like inflation, dark matter, etc; there is even an important branch of study called brane theory or braneworld cosmology.

Among the more recent models that challenge standard theories, there are multidimensional models [89], in particular the theories of strings, which were developed in the 20's and today are an area of very strong research that has resulted in the M theory: a multidimensional theory that encompasses all the others as special cases (See [6] for an excellent review). The strings, initially proposed as fundamental constituents of matter, are a particular case of p-branes (which are more general objects in p dimensions) in such a way that a 1-brane is a fundamental particle, while a 3-brane is an extended object, where the 1-branes may be limited. Under some specific hypotheses, M theory must be reduced to a five dimensional theory [8], where our universe can be seen as a 3-brane moving in a space of 5 dimensions where the fifth dimension is compactified on an orbifold.

Braneworlds in 5 dimensions are the most attractive extension of M-theory and have been studied from a cosmological point of view as an excellent approach to solve the unsolved problems in standard theory since they provide an extra ingredient in the dynamical equations at high energies and consequently in the early universe. This present thesis is an attempt to explain the inflation mech-

[1] In cosmology the term "brane" is used to refer to objects similar to the four-dimensional universe that move in a "bulk" of higher dimension. Standard Model particles are confined to such a brane.

anism by suggesting the universe is 5-dimensional with two branes: hidden and visible, and the source of inflation lives in the hidden one.

The motivation of the work in this present thesis, is the braneworld theory and the possibility of obtaining an inflationary early universe from it. Recently, there has been considerable interest in the dynamics of brane interactions. The interest was motivated partly by the insights which static brane configurations have already given to long-standing low-energy issues like the hierarchy problem, and partly by the potential application of brane collision/annihilation processes to the cosmology of the very early universe [21],[39]. In electrodynamics there is a well known tool for solving problems of electrostatic potentials, called method of images, which provides that, under certain settings, an electrostatic distribution behaves as a single charged point and viceversa [34]. The basis of this method is the principle of unicity of potential: where two potentials simultaneously satisfy Poisson's equation, the two are equivalent. In this regard, it is established that from the phenomenological point of view, two or more different phenomena can have a measurable same result. Under this philosophy, branes models propose the existence of a field of matter at the beginning and the end of a fifth coordinate, that satisfies Einstein's equations, and the effects of having distributions of matter in a hidden area, are equivalent to those produced by dark matter and dark energy in the early times of the universe. This is perhaps the main inspiration for multidimensional models: what if the visible effects around the 5-dim world are due to a distribution of matter in a hidden area? It is also is the justification for the introduction of matter and dark energy. The idea is quite attractive, especially because this opens a vast range of possible causes for the known phenomena.

The results presented in section 4.2 are the main motivation of this thesis, particularly the mechanism which generates inflation: it is not clear what is the nature of the inflaton field because it does not correspond to any physical matter field, besides that, inflation requires extremely specific initial conditions and a mechanism which stops it. Here we suppose inflation emerges as a result of having a hidden sector in the universe in which the scalar field is trapped, and its gravitational effects are detected in the visible sector. In order to sustain this hypothesis, we have solved the Einstein Equations, and have imposed boundary conditions on then. An interesting set of solutions is found which connects the cosmological evolution in both sectors. Many works in multidimensional theories point to the scalar field as been responsible for causing inflation, which is immersed in the bulk (such that models can induce either inflation or a hot big-bang radiation era, as in the ekpyrotic or cyclic scenario or in colliding bubble scenarios). Our research aims to complement such works, while opening up the possibility of investigating other possible consequences in the field of cosmology.

The present thesis is organized as follows: Chapter 2 is devoted to the introductory review to the standard cosmology and braneworld theory. It explores the main results and the motivations to introduce extra dimensions. Chapter 3 presents the main results of the study of brane dynamics, the main aspects of the RS1 model [1] and the brane cosmological equations. At the end of this chapter, we present a brief treatment of the cosmology for two branes fixed in the bulk and the master thesis results. A novel way to solve the 5D Einstein equations is presented, also showing the possible exact solutions and the corresponding constraints. Chapter 3 is devoted to the mathematical develop of the proposed model and the analysis for a universe dominated by only one component. Finally, We show the scalar field dominating case, in which the inflation phenomenon is caused by the interaction between both branes. Discussion is presented in Chapter 4.

[1]Randall–Sundrum models are models that describe the world in terms of a 5-dimensional space where the elementary particles are localized on a (3 + 1)-dimensional surface or brane. RS-1 model has a finite size for the extra dimension with two branes, one at each end

2

Modern Cosmology

Physical cosmology is the study of the universe as a whole, and is interested in knowing its physical properties: Its age, its size and shape, its limits, its components and its evolution over time, among other things. In contrast to astronomy and astrophysics, cosmology is responsible for recovering observational data from the universe at Megaparsec scales, where the particular objects of study are clusters and superclusters of galaxies [78, 84]. On scales bigger than $100Mpc$ the distribution of matter in the universe is known to be very homogeneous, both from direct observation of the galaxies and from the isotropy of the microwave background [56]. Traditionally, cosmology focused only on the study of the universe at large scales, but recent research suggests its familiarity with particle physics study at Planck time scale [59], so that for complete understanding, modern theories of quantum physics and field theory must be used.

Physical cosmology, as an observational/experimental science, is relatively new, with its beginning exactly a century ago, when Albert Einstein established the mathematical foundations of general relativity in 1915, from which the theoretical models of the universe unfolded. Einstein himself was the first to propose a static model of the universe in 1917, which included a constant term (cosmological constant) that maintained a balance with the gravitational forces of stars [35][1]. In the same year, Willem de Sitter proposed, under a similar idea, a model

[1] The cosmological constant was initially introduced by Einstein in 1917 to achieve a static universe, which coincided with the conception of the universe at that time. His original equations from 1915 did not allow for a static universe: gravity leads from a universe initially in dynamic equilibrium to one in contraction. The effect of introducing a cosmological constant

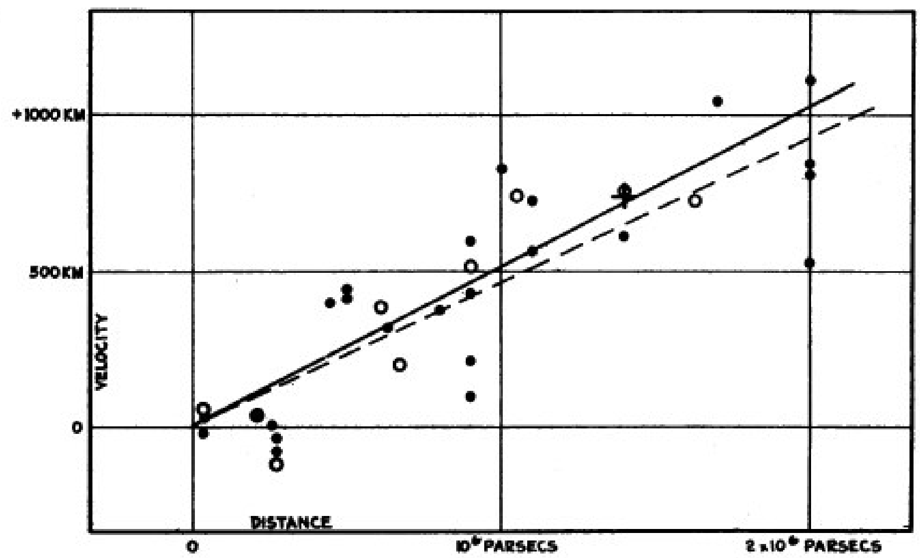

FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Figure 2.1 Original of the graphic presented by Edwin Hubble about his law of distance against radial velocity between extra-galactic nebulaes [32]

of the universe in expansion, but it did not became relevant until some time later; de Sitter's universe also contained the cosmological constant, but no other matter [66]. In 1922-1924, Alexander Friedmann [25, 26] proposed the first solutions to Einstein's equations for an expanding universe, where the possibility of having three different curvatures arises: positive, zero and negative. It was not until 1929 that Edwin Hubble published an analysis of radial velocity, with respect to Earth, of some nebulae, from which he concluded his famous Hubble law: there is a relationship between the distance of the nebula and its rate of recession [32]. From that moment on, the first successful ideas about the origin of the universe emerged. Lematrie, in 1931, concluded that if galaxies are moving away from each other, then at some point they had to be very close and confined in a highly dense region, hypothesis that he called the primeval atom or the Cosmic Egg [45]. From this idea, the term Big Bang was born, coined by Sir Fred Hoyle derisively to refer to the theory of Lematrie. It was not until 1965 that astronomers Arno

leads to a positive energy density but with negative pressure, which counteracts the attraction force of the stars.

A. Penzias and Robert W. Wilson [68] detected a 2.7 K radiation that permeated the universe; with this discovery, the theory of the Big Bang got its first experimental evidence.

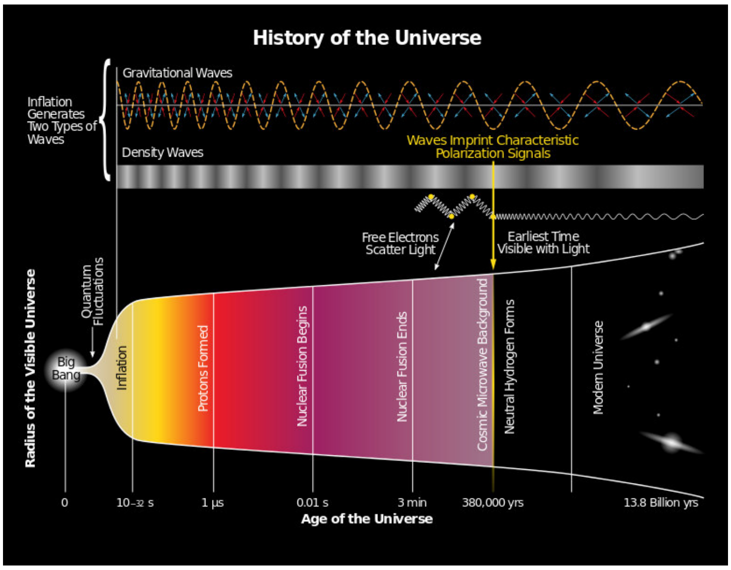

Figure 2.2 Picture of the evolution of the universe. From left to right, the figure represents the most important stages in the formation of the universe, from the Big Bang to the formation of galaxies. Image taken from https://es.m.wikipedia.org/wiki/Archivo:HistoryOfUniverse-BICEP2-20140317.png

With the Big Bang theory, the correct abundance of helium, deuterium and lithium was predicted (1966-1974), however other problems plagued cosmologists: the horizon problem, the flatness of monopoles and the formation of structure. In 1981, Alan Guth proposed the solution of the inflationary Big Bang, which solves the above problems [30] (See [85] for a review). The observational evidence for the Big Bang theory has been provided by the COBE (1990) [9] Boomerang (1998, 2003) [58], CBI (2002-2003) [75] and WMAP (2005-2009) [43] satellites, as well as other important information, such as the existence and abundance of matter and dark energy, which are detailed below.

2.1 Einstein's equations

Undoubtedly, one cannot speak of modern cosmology without taking into account the foundations of such a discipline, that is, Einstein's equations. In Einstein's general relativity theory, spacetime is curved due to the presence of a distribution of mass-energy. Such curvature is a property of spacetime itself. Thus, the acceleration of an object in a gravitational field is independent of the mass and physical condition of the body, which follows a geodesic path. There is, therefore, no difference between inertial and gravitational mass because the warping of spacetime is the same independently of the test particle used to measure [1].

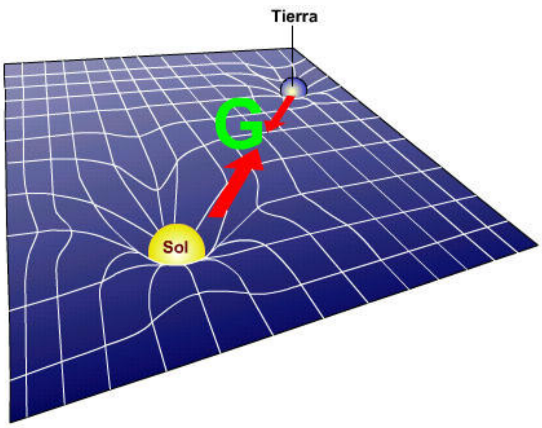

Figure 2.3 In the presence of a massive object, spacetime is curved. Image taken from https://es.wikipedia.org/wiki/Curvatura_del_espacio-tiempo

The theory of general relativity establishes the equivalence between matter and curvature through the field equations,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}, \tag{2.1}$$

where $g_{\mu\nu}$ is the metric tensor, which describes the spacetime geometry (4-

[1]While it was known from Newtonian mechanics that inertial and gravitational mass have the same magnitude, it was not until the formulation of general relativity, that Einstein offered a satisfactory explanation, as he relates: one and the same property of the body is manifested either as inertial mass, or as gravitational mass [23].

dimensional semi-Riemannian manifold [1]); R_{ij} is the Ricci's tensor[2], which describes the curvature of the metric manifold and R its contraction; κ is a numerical factor which depends on Newton's gravitational constant, G, obtained by extrapolating the Newtonian limit of equations; $T_{\mu\nu}$ is the stress-energy tensor, which describes the flow of energy and momentum and satisfies the continuity equation,

$$\nabla_\nu T^{\mu\nu} = 0. \tag{2.2}$$

The Einstein field equations can be derived using the Einstein-Hilbert action by the principle of least action,

$$S = \int \left(\frac{1}{2\kappa} R + L_M \right) \sqrt{-g} d^4x, \tag{2.3}$$

where L_M is a Lagrangian describing any field that is present in the theory. In [80], [11], [29] and [12] different solutions to Einstein's equations are displayed.

In the case for a perfect fluid, as it is considered for the universe to cosmological scales[3], we have as stress energy tensor,

$$T_{\mu\nu} = (\epsilon + p) U_\mu U_\nu + p g_{\mu\nu}, \tag{2.4}$$

where ϵ, p and U_i are the energy density, pressure and the velocity four-vector respectively.

Finally, it is necessary to consider that Einstein, motivated by the idea of a static universe, introduced into his equations an additional term called the cosmological constant (Λ), which modifies the field equations to the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}. \tag{2.5}$$

Initially, the cosmological constant drove to an unstable universe, so Einstein refers to it as a serious detriment of the formal beauty of the theory; but now, following the observations, this term is needed to describe an expanding universe. The physical nature of the cosmological constant is still a mystery, and it is only associated with the energy density of the vacuum.

[1] Srtictly speaking $g_{\mu\nu} = \partial_\mu \partial_\nu$ $\mu, \nu = 0, 1, 2, 3$, where the ∂_i are the basis of vectors for the tangent space of the manifold.

[2] The Ricci tensor is defined using Riemann tensor, $R^{\alpha}_{\mu\beta\nu}$, through the expression: $R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu}$, and the curvature scalar, R, as $R = g^{\mu\nu} R_{\mu\nu}$

[3] Scales of roughly 100 Mpc or more

2.2 Chronology of the Big-Bang

From observations such as those made by Edwin Hubble (1929), the cosmologists have suggested that in the past the universe came from a singularity (Big Bang), of which there is strong evidence supported mainly by the study of the cosmic background radiation (CMB) initiated by Penzias and Wilson (1964). Once the universe was "born", it experienced a period of exponential expansion, called "inflation" and then enter a state of "reheating". This state of relaxation allowed in certain sectors of space (through the decay of the inflaton) the creation of the standard model particles and electromagnetic radiation, which formed stars, planets and interstellar gas. By way of summary, post-Big Bang events are presented below. It should be noted that the data are not conclusive, and differ from one author to another, because the complete quantum theory to describe processes on Planck length and timescales is still in development. For a detailed analysis of the cosmological eras, refer to [42]

Planck age (0 to 10^{-43} seconds): The four fundamental forces of nature (strong nuclear, weak nuclear, gravity and electromagnetic) coexist. There is not currently a Unified Physical Theory that can give accurate data of this stage, however, quantum mechanics states that time intervals or intervals of length less than the Planck units are meaningless.

Great Unification age (10^{-43} to 10^{-36} seconds): The universe expands and cools and the first separation of the fundamental forces arises, separating gravity form the other three. It is known as grand unification, because the physical theory describing the remaining three forces (strong nuclear, weak nuclear and electromagnetic) is known as GUT. The universe was then a high density soup of particles and antiparticles of high density; the entire mass of a galaxy cluster would fit into the space occupied by a hydrogen atom. This age comes to and end when the strong nuclear force separates the weak nuclear and electromagnetic forces.

Inflationary age (10^{-36} to 10^{-32} seconds): the early universe went through a phase of exponential expansion. This rapid accelerated expansion is proposed as a solution to the problems of flatness, homogeneity and isotropy, as well as the absence of magnetic monopoles. The most accepted mechanism that causes inflation is the presence of a scalar field (inflaton), which slides from a constant potential state V_0 (false vacuum), to a global minimum (true vacuum) where inflation ends and the field oscillates and couples to other fields to form the particles and interactions of the standard model. Inflation cools the universe from a temperature $T_{GUT} \approx 10^{28}K$ to $10^{22}K$ When inflation ends, the temperature returns to the pre-inflationary temperature; this is called *reheating* or thermalization because the large potential energy of the inflaton field decays into particles and fills the universe with Standard Model particles, including electromagnetic radiation, starting the radiation dominated phase of the universe.

Finally, there was the Baryogenesis, that is an asymmetry between the amount of particles and antiparticles created when part of the energy of the universe materialized in baryons. The Dirac equation predicts the existence of particles and antiparticles, and the CPT theorem guarantees that both particles have the same mass and half-life, but opposite charge; so the observed asymmetry (for every 10 billion particle-antiparticle pairs, there is one extra particle that does not have an antiparticle to annihilate with and become background radiation) may be due to some violation of CP symmetry after the inflationary stage.

Electroweak age (10^{-36} to 10^{-12} seconds): This stage was overlaid with inflation, or what is the same, the inflation occurred during electroweak era, where the strong interaction becomes distinct from the electroweak interaction, which is known as spontaneous symmetry breaking, and that's when the fundamental particles have acquired their mass by the Higgs mechanism.

Quarks age (10^{-12} to 10^{-6} seconds): in this period the four basic forces of nature are already in their present forms. The Universe still has such a high temperature that prevents the binding of quarks forming hadrons.

Hadrons age (10^{-6} to 1 second): the universe continues to expand, and the temperature falling. When it gets cold enough, the quark-gluon plasma formation begins with hadrons, then protons and neutrons come together to their antiparticles. Neutrinos are decoupled from the rest of the particles and begin to travel freely through the universe. If we can dispose the necessary detectors (very elusive neutrinos) could catch a cosmic background remnant of them as existing for photons.

Leptons age (1 to 10 seconds) most anti-hadrons and hadrons have been destroyed at the end of the age of hadrons and leptons now (electrons and the like) and anti-leptons dominate the mass of the universe. At the end of this age, most leptons and their antiparticles are annihilated, leaving a small residue that will be arriving at our days.

Photons age (10 seconds to 300,000 years) after the age of leptons energy of the universe is dominated by photons. The universe is so energetic that even photons interact with protons, electrons and atomic nuclei. It will take 300,000 years before the universe has cooled enough for photons to continue their path freely, as neutrinos have already done.

Nucleosynthesis (1 seconds to 3 minutes): the formation of nuclei occurs during the age of leptons and first minutes of the age of photons. In this period the protons and neutrons combine to form nuclei of hydrogen, helium and traces of lithium. Due to the expansion of the universe, after 3 minutes the temperature is not enough to follow the process and it stops. Now the universe is plasma of atomic nuclei, electrons, neutrinos and photons.

Recombination (300,000 years): the universe has continued to expand and the temperature has dropped enough to allow atomic nuclei combine with electrons and form atoms. Finally photons are free to follow their path, the universe becomes transparent to them. We can see the echo of this was in the Cosmic Microwave Background.

Reionization (150,000,000 1,000,000,000 years): tiny differences in the homogeneity of the universe, caused certain regions of it to begin to accumulate more matter due to gravity. Finally, the gravitational collapse turns the first quasars, the radiation is so intense that reionized clouds surrounding atoms. Once the star clouds began to collapse on themselves due to gravity and interaction with quasar radiation, the first stars emerged. The generation of heavier atomic elements in their centers, and later planting in the Universe by the supernova explosions, gave way to the formation of the constituent elements of life. New generations of stars lighting up the Universe continue to the present.

Galaxy formation: gravity causes matter volumes collapsing to form larger structures: Galaxies.

Today it is well known [3], with some degree of accuracy, that the universe has an age of 13.75×10^9 years, and the current visible radius is about 14 billion parsecs in addition that is currently expanding rapidly, and it is unknown the final destination (although most likely expand to cool completely). The present overall density of the Universe is very low, roughly 9.9×10^{-30} grams per cubic centimetre. The primary components of the energy density are the dark energy and dark matter, consisting of 73% dark energy, 23% cold dark matter and 4% ordinary matter, and the "dark" term is used to indicate the total ignorance of its nature, and they have been postulated due to inconsistencies between observation and theory.

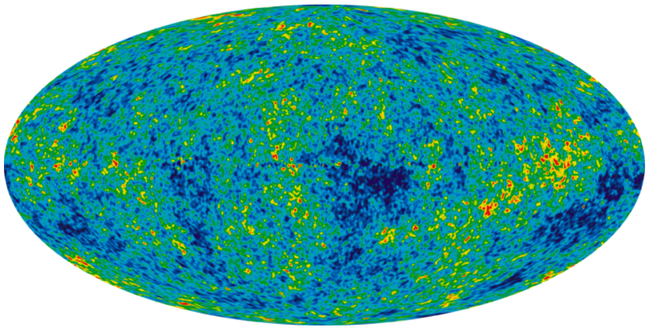

Figure 2.4 9-year WMAP image of background cosmic radiation (2012). Figure taken of `https://es.m.wikipedia.org/wiki/Archivo:WMAP_image_of_the_CMB_anisotropy.jpg`

Many of the principal knowledge of the early universe is supported by the study of the CMB (cosmic microwave background) which is the thermal radiation assumed to be a relic of the early universe (for a mini review, refers to [81]). The universe was once very hot and dense, the photons and baryons would have formed a plasma. As the universe expanded and cooled there came a point when the radiation (photons) decoupled from the matter. The radiation cooled and is now at 2.72 Kelvin and that the spectrum of the radiation is almost exactly that of a black body with a little anisotropy. The anisotropy of the cosmic microwave background is divided into two types: primary anisotropy, due to effects which occur at the last scattering surface and before; and secondary anisotropy, due to effects such as interactions of the background radiation with hot gas or gravitational potentials, which occur between the last scattering surface and the observer.

The structure of the cosmic microwave background anisotropies is principally determined by two effects: acoustic oscillations and diffusion damping. The acoustic oscillations arise because of a conflict in the photon-baryon plasma in the early universe. The pressure of the photons tends to erase anisotropies, whereas the gravitational attraction of the baryons makes them tend to collapse to form dense haloes. These two effects compete to create acoustic oscillations which give the microwave background its characteristic peak structure. The peaks correspond, roughly, to resonances in which the photons decouple when a particular mode is at its peak amplitude.

2.3 Standard Big-Bang Cosmology

Based in the cosmological principle, which means that the universe is homogeneous and isotropic on large distance, the standard cosmology have the Friedmann-Robertson-Walker (FRW) metric as the most general metric that describes the universe geometry [17]:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{2.6}$$

Where $a(t)$ is the scale factor with t being the cosmic time. The constant $k = \{+1, 0, -1\}$ is the spatial curvature, where the values correspond to closed, flat, and open universes, respectively.

In order to know the dynamical evolution of the universe, it is required to solve the Einstein equations, which we suppose are valid at large scales. The Einstein equations are expressed as [87]

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \tag{2.7}$$

where $R_{\mu\nu}$, R, $T_{\mu\nu}$, and G are the Ricci tensor, Ricci scalar, energy momentum tensor, gravitational constant, respectively. The Planck energy, $m_{\rm pl} = 1.2211 \times 10^{19}\,\text{GeV}$, is related with G through the relation $m_{\rm pl} = (\hbar c^5/G)^{1/2}$. Here $\hbar$ and c are the Planck's constant and the speed of light, respectively. Traditionally the theorist works with a natural system of unities in which $\hbar = c = 1$. Λ is a cosmological constant originally introduced by Einstein, but recently revived as a primary element that produces accelered expansion in the Λ-Cold Dark Matter model.

For the background metric (2.6) with a negligible cosmological constant, the Einstein equations (2.7) yield

$$H^2 = \frac{8\pi}{3m_{\rm pl}^2}\rho - \frac{k}{a^2}, \tag{2.8}$$

$$\dot{\rho} + 3H(\rho + p) = 0\,, \tag{2.9}$$

where a dot denotes the derivative with respect to t, and $H \equiv \dot{a}/a$ is the Hubble expansion rate. Eqs. (2.8) and (2.9) are so called the Friedmann and Fluid equations, respectively. The evolution of the universe is dependent on the material within it. This is characterized by the equation of state between the energy density and the pressure, $p(t) = \omega\rho(t)$. Typical examples are :

$$\omega = 1/3 \rightarrow p = \rho/3\,, \quad \text{radiation}\,, \tag{2.10}$$

$$\omega = 0 \rightarrow p = 0\,, \quad \text{dust}\,, \tag{2.11}$$

$$\omega = -1 \rightarrow p = -\rho\,, \quad \text{quintessence}\,. \tag{2.12}$$

When the spatial geometry is flat ($k = 0$), the solutions for Eqs. (2.8) and (2.9) are

$$\text{Radiation dominant}: \quad a \propto t^{1/2}\,, \quad \rho \propto a^{-4}\,, \tag{2.13}$$

$$\text{Dust dominant}: \quad a \propto t^{2/3}\,, \quad \rho \propto a^{-3}\,, \tag{2.14}$$

$$\text{Quintessence dominant}: \quad a \propto e^{kt}\,, \quad \rho = constant\,. \tag{2.15}$$

Combining Eqs. (2.8) and (2.9) give the well known equation of acceleration,

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3m_{\rm pl}^2}(\rho + 3p)\,. \tag{2.16}$$

In these simple cases, the universe is expanding deceleratedly ($\ddot{a} < 0$) as confirmed by Eq. (2.16). In contrast, a positive $\ddot{a}$ is possible only when $\rho+3p < 0$ is satisfied

The Friedmann equation (2.8) can be rewritten as

$$\Omega - 1 = \frac{k}{a^2H^2}\,, \tag{2.17}$$

where

$$\Omega \equiv \frac{\rho}{\rho_c}, \qquad \text{with} \qquad \rho_c \equiv \frac{3H^2m_{\rm pl}^2}{8\pi}\,. \tag{2.18}$$

Here the density parameter Ω is the ratio of the energy density to the critical density. Current observations suggest $\Omega - 1 \approx 0$ and a positive accelerated universe, which suggest the Λ term is not negligible at all.

Although today there is a well-established theory that explain to a high accuracy the physics of the visible universe, there are also other non-solved issues, including inflation, which is mentioned in the next section.

2.4 Composition of the Universe

Very little has been said until now about the primary components of the universe, but much of the study of the cosmos has to do with the kind of matter that composes it. The term $\Omega = \rho/\rho_c$ that appears in the equation (2.17), encompasses all types of matter-energy contained in the universe [83]. Recent observations suggest a value $\Omega \approx 1$, that is, that the universe is nearly flat, $k \approx 0$. This result suggests that the total energy density in the universe must be close to the critical density, that is, $\rho \approx \rho_c$. However, experimental calculations suggest that baryon matter and radiation only contribute a small fraction of Ω, around 0.04, while 0.23 exists in the form of dark matter and the remaining 0.73 in the form of dark energy. Table (2.1) shows a complete inventory of all the components in the universe.

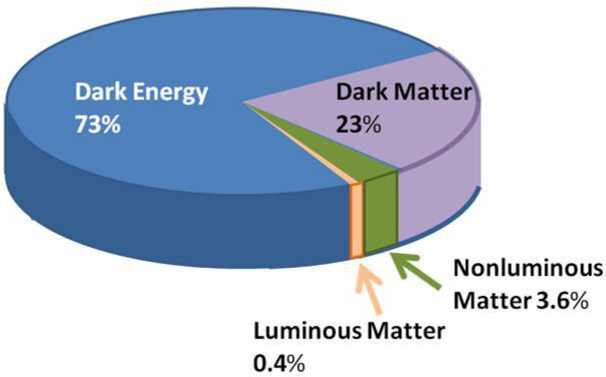

Figure 2.5 Recent observations point towards the Universe is mostly composed of dark energy and dark matter, both of which are not well understood at present. Less than 5% of the Universe is ordinary matter,which is present in form of Star, radiation and interstellar gas. Image taken from `https://es.m.wikipedia.org/wiki/Archivo:Matter_Distribution.JPG`

According to the Friedmann equations, the expansion rate of the universe is determined by the energy density and also by the equation of state of its constituents. The main components of the matter composition that play an important role at temperatures below a few MeV are primordial radiation, baryons, electrons, neutrinos, dark matter and dark energy.

Primordial radiation. The cosmic microwave background (CMB) radiation has temperature $T_{\gamma 0} \approx 2.73K$. Its current energy density is about $\rho_{\gamma 0} \approx 10^{34} gcm^{-3}$ and constitutes only 10^{-5} of the total energy density. The radiation has a perfect Planckian spectrum and appears to have been present in the very early universe at energies well above one GeV. Since the temperature of radiation varies in inverse proportion to the scale factor, it must have been very high in the past.

Baryonic matter. This is the material out of which the planets, stars, clouds of gas and possibly dark stars of low mass are made [27]; some of it could also form black holes. We will see later that the data on light element abundances and CMB fluctuations clearly indicate that the baryonic component contributes only a small percentage of the critical energy density ($\Omega_b \approx 0.04$). The number of photons per baryon is of the order of 10^9.

Table 2.1. The Cosmic Energy Inventory [27]

				Components[a]	Totals[a]
1	dark sector				0.954 ± 0.003
1.1	dark energy			0.72 ± 0.03	
1.2	dark matter			0.23 ± 0.03	
1.3	primeval gravitational waves			$\leq 10^{-10}$	
2	primeval thermal remnants				0.0010 ± 0.0005
2.1	electromagnetic radiation			$10^{-4.3\pm0.0}$	
2.2	neutrinos			$10^{-2.9\pm0.1}$	
2.3	prestellar nuclear binding energy			$-10^{-4.1\pm0.0}$	
3	baryon rest mass				0.045 ± 0.003
3.1	warm intergalactic plasma			0.040 ± 0.003	
3.1a	virialized regions of galaxies		0.024 ± 0.005		
3.1b	intergalactic		0.016 ± 0.005		
3.2	intracluster plasma			0.0018 ± 0.0007	
3.3	main sequence stars	spheroids and bulges		0.0015 ± 0.0004	
3.4		disks and irregulars		0.00055 ± 0.00014	
3.5	white dwarfs			0.00036 ± 0.00008	
3.6	neutron stars			0.00005 ± 0.00002	
3.7	black holes			0.00007 ± 0.00002	
3.8	substellar objects			0.00014 ± 0.00007	
3.9	HI + HeI			0.00062 ± 0.00010	
3.10	molecular gas			0.00016 ± 0.00006	
3.11	planets			10^{-6}	
3.12	condensed matter			$10^{-5.6\pm0.3}$	
3.13	sequestered in massive black holes			$10^{-5.4}(1+\epsilon_n)$	
4	primeval gravitational binding energy				$-10^{-6.1\pm0.1}$
4.1	virialized halos of galaxies			$-10^{-7.2}$	
4.2	clusters			$-10^{-6.9}$	
4.3	large-scale structure			$-10^{-6.2}$	
5	binding energy from dissipative gravitational settling				$-10^{-4.9}$
5.1	baryon-dominated parts of galaxies			$-10^{-8.8\pm0.3}$	
5.2	main sequence stars and substellar objects			$-10^{-8.1}$	
5.3	white dwarfs			$-10^{-7.4}$	
5.4	neutron stars			$-10^{-5.2}$	
5.5	stellar mass black holes			$-10^{-4.2}\epsilon_s$	
5.6	galactic nuclei	early type		$-10^{-5.6}\epsilon_n$	
5.7		late type		$-10^{-5.8}\epsilon_n$	
6.	poststellar nuclear binding energy				$-10^{-5.2}$
6.1	main sequence stars and substellar objects			$-10^{-5.8}$	
6.2	diffuse material in galaxies			$-10^{-6.5}$	
6.3	white dwarfs			$-10^{-5.6}$	
6.4	clusters			$-10^{-6.5}$	
6.5	intergalactic			$-10^{-6.2\pm0.5}$	
7	poststellar radiation				$10^{-5.7\pm0.1}$

Table 2.1 (cont'd)

			Components[a]	Totals[a]
7.1	resolved radio-microwave		$10^{-10.3\pm0.3}$	
7.2	far infrared		$10^{-6.1}$	
7.3	optical		$10^{-5.8\pm0.2}$	
7.4	X-γ ray		$10^{-7.9\pm0.2}$	
7.5	gravitational radiation	stellar mass binaries	$10^{-9\pm1}$	
7.6		massive black holes	$10^{-7.5\pm0.5}$	
8	stellar neutrinos			$10^{-5.5}$
8.1	nuclear burning		$10^{-6.8}$	
8.2	white dwarf formation		$10^{-7.7}$	
8.3	core collapse		$10^{-5.5}$	
9	cosmic rays and magnetic fields			$10^{-8.3^{+0.6}_{-0.3}}$
10	kinetic energy in the intergalactic medium			$10^{-8.0\pm0.3}$

[a]Based on Hubble parameter $h = 0.7$.

Dark matter and dark energy. The CMB fluctuations imply that at present the total energy density is equal to the critical density. This means that the largest fraction of the energy density of the universe is dark and nonbaryonic. It is not quite clear what constitutes this dark component. The first hypothesis to postulate dark matter based upon robust evidence was Vera Rubin's in the 1960s and 1970s, using galaxy rotation curves [79]. Combining the data on CMB, large scale structure, gravitational lensing and high-redshift supernovae, it appears that the dark component is a mixture of two or more constituents. More precisely, it is composed of cold dark matter and dark energy. The cold dark matter has zero pressure and can cluster, contributing to gravitational instability. Various (supersymmetric) particle theories provide us with natural candidates for the cold dark matter, among which weakly interacting massive particles are most favored at present. The non-baryonic cold dark matter contributes only about 25% of the critical density. The remaining 70% of the missing density comes in the form of nonclustered dark energy with negative pressure. It may be either a cosmological constant ($p = -\rho$) or a scalar field (quintessence) with $p = \omega\rho$, where ω is less than $-1/3$ today.

Primordial neutrinos. These are an inevitable remnant of a hot universe.

Figure 2.6 Strong gravitational lensing as observed by the Hubble Space Telescope in Abell 1689 indicates the presence of dark matter. Image taken from `https://esahubble.org/images/heic1317a/`

If the three known neutrino species were massless, their temperature today would be $T_\nu \approx 1.9K$ and they would contribute 0.68 times the radiation density. Atmospheric neutrino oscillation experiments suggest that the neutrinos have small masses. Even so, it appears that they cannot constitute more than 1% of the critical density.

2.5 Inflation

Inflation is the mechanism by which the universe expands exponentially and has been postulated as the response to various unsolved problems in standard cosmology, namely the flatness problem (why the universe is nearly flat) horizon (why the universe has the same temperature in all directions) and magnetic monopoles (why no magnetic monopoles have been detected yet). Although the mechanism that generates inflation is unknown, in 2014 the project BICEP2 [2] announced the detection of gravitational waves from inflation, providing a strong support to the inflationary theory. The basic ideas of inflation were originally proposed by Alan Guth in 1981, which is now known as old inflation. This corresponds to the de-Sitter inflation which makes use of the first-order transition to true vacuum.

However, it has a serious shortcoming in that the universe becomes inhomogeneous by the bubble collision soon after the inflation ends. Another version was proposed by Andrei Linde and Albrecht and Steinhardt in 1982, which is known as new inflation [49, 50]. This corresponds to the slow-roll inflation with the second-order transition to true vacuum. Unfortunately this scenario also suffers from a fine-tuning problem of spending enough time in false vacuum to lead to a sufficient amount of inflation. In 1983 Linde [48] considered the variant version of the slow-roll inflation called chaotic inflation, in which initial conditions of scalar fields are chaotic. According to this model, our homogeneous and isotropic universe may be produced in the regions where inflation occurs sufficiently.

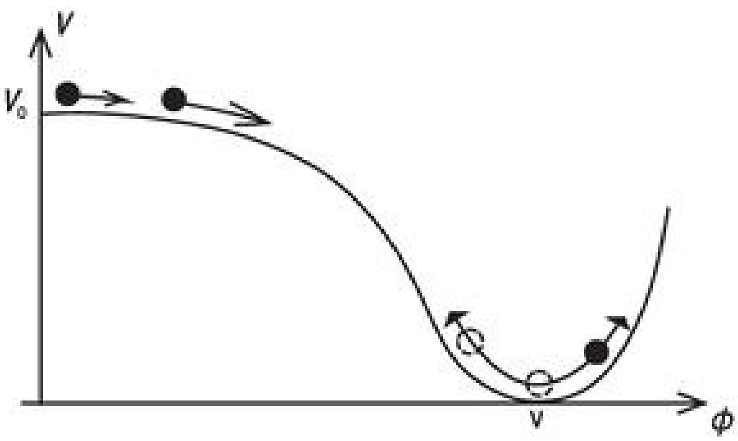

Figure 2.7 Shape of the Potential $V(\phi)$. The scalar field rolls slowly until reach the minimum of potential V_0. Image taken from `http://universeinproblems.com/index.php/File:Inflation_3_2_1.jpg`

As a first approach, inflation is modeled by the existence of a real scalar field which filled the early universe. This scalar field is subject to the action of a potential which has a specific shape such that slow-roll conditions are satisfied: it must be flat enough to generate the exact amount of inflation. At present, while inflation is understood principally by its detailed predictions of the initial conditions for the hot early universe, the particle physics is largely ad hoc modeling. As such, though predictions of inflation have been consistent with the results of observational tests, there are many open questions about the theory.

The scalar fields are the main ingredients in particle physics theories and, more recently, in the early universe models. Consider a homogeneous scalar field ϕ, (inflaton), whose potential energy leads to the exponential expansion of the universe (For a review, refers to [16]). The energy density and the pressure density of the inflaton can be described, respectively, as

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,, \tag{2.19}$$

where $V(\phi)$ is the potential of the inflaton. Substituting Eq. (2.19) for Eqs. (2.8) and (2.9), it gets

$$H^2 = \frac{8\pi}{3m_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]\,, \tag{2.20}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,, \tag{2.21}$$

where $\kappa^2 \equiv 8\pi G = 8\pi/m_{\rm pl}^2$, and the curvature term k/a^2 in Eq. (2.8) has been neglected. During inflation, the relation $\rho + 3p < 0$ is satisfied and yields $\dot{\phi}^2 < V(\phi)$, which indicates that the potential energy of the inflaton dominates over the kinetic energy of it. Therefore a flat potential of the inflaton is required in order to lead to sufficient amount of inflation. Imposing the slow-roll conditions: $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll 3H\dot{\phi}$, Eqs. (2.20) and (2.21) are approximately given as

$$H^2 \simeq \frac{8\pi}{3m_{\rm pl}^2}V(\phi), \tag{2.22}$$

$$3H\dot{\phi} \simeq -V'(\phi). \tag{2.23}$$

Defining the so-called slow-roll parameters

$$\epsilon \equiv \frac{m_{\rm pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2\,, \quad \eta \equiv \frac{m_{\rm pl}^2}{8\pi}\frac{V''}{V}\,, \tag{2.24}$$

It can easily verify that the above slow-roll approximations are valid when

$$\epsilon \ll 1, \quad |\eta| \ll 1\,. \tag{2.25}$$

The inflationary phase ends when ϵ and $|\eta|$ grow of order unity. A useful quantity to describe the amount of inflation is the number of e-foldings, defined by

$$N \equiv \ln\frac{a_f}{a_i} = \int_{t_i}^{t_f} H dt\,, \tag{2.26}$$

where the subscripts i and f denote the quantities at the beginning and the end of the inflation, respectively.

In order to solve the flatness problem, Ω is required to be $|\Omega_f - 1| \sim 10^{-60}$ right after the end of inflation. Meanwhile the ratio $|\Omega - 1|$ between the initial and final phase of inflation is given by

$$\frac{|\Omega_f - 1|}{|\Omega_i - 1|} \simeq \left(\frac{a_i}{a_f}\right)^2 = e^{-2N}, \tag{2.27}$$

where is used the fact that H is nearly constant during inflation. Assuming that $|\Omega_i - 1|$ is of order unity, the number of e-foldings is required to be $N \sim 70$ to solve the flatness problem. A similar number of e-foldings are required to solve the horizon problem.

Inflationary cosmology solves a considerable number of cosmological problems such as flatness, horizon, and monopole problems. In addition, inflation makes it possible to generate nearly scale-invariant density perturbations, which is consistent with observations. Inflation is really an efficient mechanism to solve the cosmological problems associated with standard big-bang cosmology. In addition, elementary particles can be produced during the reheating stage after inflation through the decay of the inflaton. It is fair to say that standard inflation with a slow-roll flat potential is the most promising scenario of the very early universe among the models proposed so far. Nevertheless there are still many unsolved problems even in the inflationary cosmology: What is the origin of the inflaton field? What is the state of the universe before inflation? Can the initial singularity be avoided?

2.6 CMB

The energy content in radiation from beyond our Galaxy is dominated by the Cosmic Microwave Background (CMB), discovered in 1965 [68]. The spectrum of the CMB is well described by a blackbody function with $T = 2.725K$. This spectral form is one of the main pillars of the hot Big Bang model for the early universe. The lack of any observed deviations from a blackbody spectrum constrains physical processes over cosmic history at redshifts $z \leq 10^7$. However, at the moment, all viable cosmological models predict a very nearly Planckian spectrum, and so they are not stringently limited.

Another observable quantity inherent in the CMB is the variation in temperature (or intensity) from one part of the microwave sky to another. Since the first detection of these anisotropies by the COBE satellite [9], there has been intense activity to map the sky at increasing levels of sensitivity and angular resolution. A series of ground- and balloon-based measurements was joined in 2003 by the first results from NASAs Wilkinson Microwave Anisotropy Probe (WMAP) [10]. These observations have led to a stunning confirmation of the Standard Model of Cosmology. In combination with other astrophysical data, the CMB anisotropy measurements place quite precise constraints on a number of cosmological parameters, and have launched us into an era of precision cosmology.

Observations show that the CMB contains anisotropies at the 10^{-5} level, over a wide range of angular scales. These anisotropies are usually expressed by using a spherical harmonic expansion of the CMB sky:

$$T(\theta, \phi) = \sum_{lm} a_{lm} Ylm(\theta, \phi) \tag{2.28}$$

The vast majority of the cosmological information is contained in the temperature 2 point function, i.e., the variance as a function of separation θ. Equivalently, the power per unit lnl is $l \sum_m \|a_{lm}\|^2/4\pi$.

The CMB has a mean temperature of $T_\gamma = 2.72548 \pm 0.00057K$ (1σ) [24], which can be considered as the monopole component of CMB maps, a_{00}. Since all mapping experiments involve difference measurements, they are insensitive to this average level. Monopole measurements can only be made with absolute temperature devices, such as the FIRAS instrument on the COBE satellite [9]. Such measurements of the spectrum are consistent with a blackbody distribution over more than three decades in frequency. A blackbody of the measured temperature corresponds to $n_\gamma = (2\zeta(3)/\pi^2)T_\gamma^3 \simeq 411cm^{-3}$ and $\rho_\gamma = (\pi^2/15)T^4 \simeq 4.64 \times 10^{-34} gcm^{-3} \simeq 0.260eVcm^{-3}$.

The largest anisotropy is in the $l = 1$ (dipole) first spherical harmonic, with amplitude $3.346 \pm 0.017mK$. The dipole is interpreted to be the result of the Doppler shift caused by the solar system motion relative to the nearly isotropic blackbody field, as confirmed by measurements of the radial velocities of local galaxies.

Excess variance in CMB maps at higher multipoles ($l \geq 2$) is interpreted as being the result of perturbations in the energy density of the early Universe, manifesting themselves at the epoch of the last scattering of the CMB photons. In the hot Big Bang picture, this happens at a redshift $z = 1100$, with little dependence on the details of the model. The process by which the hydrogen and helium nuclei can hold on to their electrons is usually referred to as recombination. Before this epoch, the CMB photons are tightly coupled to the baryons, while afterwards they can freely stream towards us

2.7 The hierarchy problem

Other strong motivations for the study of branes theory, which is the subject in this thesis, is the fundamental problem of the hierarchies in particle physics [36]. In a four-dimensional world there are at least two fundamental energy scales: the weak scale, $m_{EW} \sim 10^3$ GeV and the Planck scale, $m_{Pl} \sim 10^{19}$ GeV. Physics is well described by the Standard Model at least up to 100 GeV or so. At the Planck scale, gravity becomes as strong as the SM interactions and a quantum theory of gravity is required [37]. Why is there such a vast difference between the two scales? This question is the essence of the *hierarchy problem.* Consider the Higgs boson whose physical mass, $m_H \sim m_{EW}$. Now suppose our theory is cut-off at some large scale $\Lambda \sim m_{Pl}$, where obviously $m_H \ll \Lambda$. Calculations of the *one loop* correction for the Higgs mass leads to the equality $m_H^2 = m_0^2 + \Lambda^2$. The bare mass, m_0^2, must then be of order $-\Lambda^2$ to give a renormalised mass near the weak scale, but this implies a surprising *fine tuning.* If we believe that our fundamental theory contains scales as high as the Planck scale, then the cancellation just described is disturbingly precise, given the huge numbers involved. What is more, this bizarre precision is required again at all subsequent orders of perturbation theory.

Traditionally, it is thought that this vast desert between the weak and the Planck scales must be populated with new theories, such as supersymmetry [61]. Above the scale of supersymmetry breaking, the problems with radiative corrections to the Higgs mass are solved, although we may still ask why the desert exists at all.

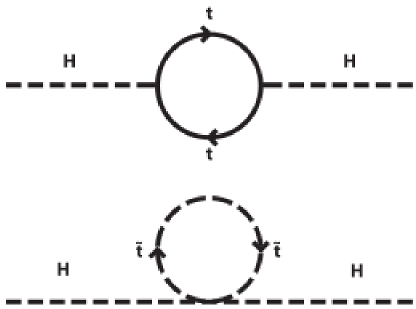

Figure 2.8 One loop corrections to the Higgs Mass. Image taken from https://en.wikipedia.org/wiki/Hierarchy_problem

2.8 M-theory and Branes

Brane-world models are an interesting viewpoint of the universe dynamics adding new degrees of freedom which can help to solve the problems of dark matter(DM) and dark energy (DE). In principle, these models has been motivated by String theory and M-theory, where our visible universe can be seen as a 4D manifold (brane) immersed in a space-time of more than three spatial dimensions (bulk) where usually the standard model of particles (SM) fields are trapped on the brane, being the gravity the only field that can escape to the bulk.

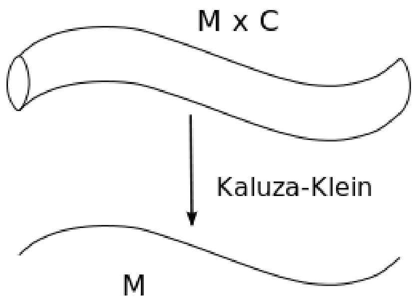

Figure 2.9 Kaluza-Klein compactification. Image taken form https://es.wikipedia.org/wiki/Compactacion_(fisica)

Historically, the theories with more spatial dimensions begin with the Kaluza and Klein works, who's, following the Nordstrom idea, built a 5D theory as an attempt for unify gravity and electromagnetic forces. A novel feature of 5D models is that m_{Pl}, the 4D Planck scale, is not more the fundamental scale, which is M_5; additionally, the compact extra dimensions implies every multi-dimensional field corresponds to a Kaluza-Klein tower of four-dimensional particles with increasing masses. At low energies, only massless (at $E >> 1/R$) particles can be produced, whereas at $E \approx 1/R$ extra dimensions are detectable. This is the starting point of string theories which, trying to reconcile quantum mechanics and general relativity, postulate a more general space-time of 4+D dimensions, where the fundamental particles are conceived as small vibrating strings. Recent research in string theory and its generalization M-theory have suggested the number of dimensions, to make a consistent quantum string theory is eleven. Inherited in these models are the p-branes ($0 < p < 9$), which are the fundamental constituents of the universe. A brane, where the open strings have their endpoints, is called D-brane. Our visible universe can be a very large D-brane extending over three spatial dimensions. Material objects, made of open strings, are confined on such D-brane, while gravity and other exotic matter such as the dilaton can propagate in the bulk. This scenario is called brane cosmology or brane-world cosmology. A reduction to 5D of M-theory is suggested by Horava and Witten [8, 31].

The strong coupling limit of the $E_8 \times E_8$ heterotic string theory at low energy is described by 11D supergravity with the eleventh dimension compactifed on an S_1/Z_2 orbifold. The two boundaries of space-time are two 10-branes, on which gauge theories are confined. Witten argued that 6 of the 11 dimensions can be consistently compactifed on a CalabiYau threefold and that the size of the Calabi-Yau manifold can be substantially smaller than the space between the two boundary branes. Thus, in that limit space-time looks five-dimensonal. A 5D realization of the HW theory and the corresponding brane-world cosmology is given in [53, 54, 55]. These solutions can be thought of as efectively 5D, with an extra dimension that can be large relative to the fundamental scale, providing the basis for the Arkani-Dimopoulos-Dvali (ADD) [7], Randall-Sundrum (RS) [76, 77], and Dvali-Gabadadze-Porrati (DGP) brane models of 5D gravity [20].

2.8.1 ADD model

Another solution to solve the hierarchy problem that is radically different to supersymmetry, is known as ADD model. The model assumes that there is only one fundamental energy scale, the weak scale. The observed large Planck scale comes from extra dimensions, beyond the traditional four. As observers, we are limited to a braneworld embedded in a $(4 + n)$-dimensional space-time. The $(4 + n)$-dimensional Planck scale, M, is now the fundamental scale of gravity, and is taken to be of order the weak scale. The extra dimensions are given by an n-dimensional compact space of volume R^n. In order to recover standard gravitational behavior, $V(r) \propto 1/r$, our effective four-dimensional Planck scale is given by

$$m_{Pl}^2 = M^{n+2} R^n. \tag{2.29}$$

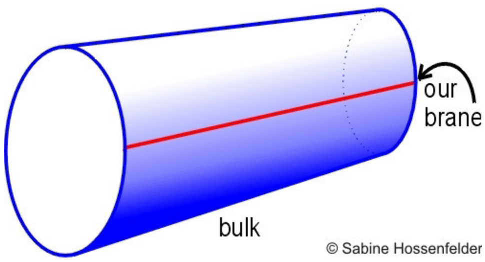

Figure 2.10 Figure illustrates the whole (4+n) dimensional universe, where the brane is placed. Additional n dimensions are compact. Figure taken form `http://backreaction.blogspot.com/2006/07/extra-dimensions.html`

By taking R^n to be sufficiently large we can recover $m_{pl} \sim 10^{19}$ GeV. However, in some sense the hierarchy problem has not be solved, because there is now a new hierarchy between the weak scale and the compactification scale, $1/R^n \ll m_{EW}$. Fortunately, the Randall-Sundrum I (RS1) model is an extension of these ideas that does not appear to transfer the problem in this way. Other problem for the model is the lack of experimental evidence; results from the Large Hadron Collider (2014) and Fermi-LAT do not appear to support the model thus far [1],[18].

2.8.2 Randall-Sundrumm overview

As we explained before, originally the brane-world models attempted to solve the hierarchy problem, but the ADD only translate the hierarchy from one side to another. In the Randall Sundrum I (RSI) model, the mechanism is completely diferent [40]. Instead of using large dimensions, RS used the warped factor $\sigma(y) = k\,|y|$, for which the mass m_0 measured on the invisible (Planck) brane is related to the mass m measured on the visible (TeV) brane by $m = e^{-ky_c} m_0$. Clearly, by properly choosing the distance y_c between the two branes, one can lower m to the order of TeV, even m_0 is still in the order of M_{pl}. It should be noted that the five-dimensional Planck mass M_5 in the RS1 scenario is still of the order of M_{pl} and the two are related by $M_{pl}^2 = M^3 k^{-1}(1 - e^{-2ky_c}) \simeq M_5^2$ for $k \simeq M_5$.

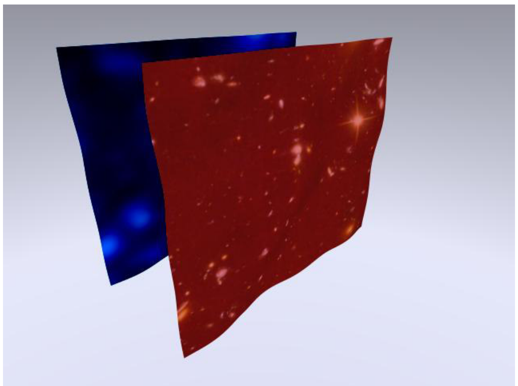

Figure 2.11 Two parallel branes. Image taken from `https://www.infoniac.ru/news/Nauka-v-fil-me-Interstellar-krotovye-nory-chernye-dyry-prostranstvo-vremy html`

The RS brane-worlds and their generalizations (to include matter on the brane, scalar fields in the bulk, etc.) provide phenomenological models that reflect at least some of the features of M theory, and that bring exciting new geometric and particle physics ideas into play. The RS models also provide a framework for exploring holographic ideas that have emerged in M theory. Roughly speaking, holography suggests that higher-dimensional gravitational dynamics may be determined from knowledge of the fields on a lower-dimensional boundary. The AdS/CFT correspondence is an example, in which the classical dynamics of the higher-dimensional gravitational field are equivalent to the quantum dynamics of a conformal field theory (CFT) on the boundary.

2.8.3 DGP model

The Dvali-Gabadadze-Porrati braneworld model has been considered as a model which could modify gravity because of the existence of the extra-dimensions. In the DGP model the 3-brane is embedded in a Minkowski bulk spacetime with infinitely large 5th extra dimensions. The Newton's law can be recovered by adding a 4-dimensional (4D) Einstein-Hilbert action sourced by the brane curvature to the 5D action [7]. While the DGP model recovers the standard 4D gravity for small distances, the effect from the 5D gravity manifests itself for large distances. Remarkably it is possible to realize the late-time cosmic acceleration without introducing an exotic matter source. The DGP model is given by the action

$$S = \frac{1}{2\kappa_{(5)}^2}\int d^5X\sqrt{-\tilde{g}}\tilde{R} + \frac{1}{2\kappa_{(4)}^2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_M \tag{2.30}$$

where $\tilde{g}_{AB}$ is the metric in the 5D bulk and $g_{\mu\nu} = \partial_\mu X^A \partial_\mu X^B \tilde{g}_{AB}$ is the induced metric on the brane with $X^A(x^c)$ being the coordinates of an event on the brane labeled by x^c. The first and second terms in the above equation correspond to Einstein Hilbert actions in the 5D bulk and on the brane, respectively. Note that $\kappa_{(5)}^2 = 1/M_{(5)}^3$ and $\kappa_{(4)}^2 = 1/M_{(4)}^2$ are 5D and 4D gravitational constants, respectively. The Lagrangian brane L_M describes matter localized on the 3-brane. Solving Einstein equations, with aid of the Israel junction conditions [60], we obtain the modified Friedmann equation for a FLRW brane:

$$H^2 - \frac{\epsilon}{r_c}H = \frac{\kappa_{(4)}^2}{3}\rho_M \tag{2.31}$$

where $\epsilon = \pm$, H and ρ_M are the Hubble parameter and the matter energy density on the brane respectively. In the DGP the length scale is given by r_c, defined by

$$r_c = \frac{M_{(4)}^2}{2M_{(5)}^3}. \tag{2.32}$$

In the regime $r_c \gg H^{-1}$ the first term dominates and the standard Friedmann equation is recovered. Meanwhile, in the regime $r_c \ll H^{-1}$, the second term leads to a modification to the standard Friedmann equation. If $\epsilon = 1$, there is a de Sitter solution characterized by $H_{dS} = 1/r_c$. One can realize the cosmic acceleration today if r_c is of the order of the present Hubble radius H_0^{-1}. This self acceleration is the result of gravitational leakage into extra dimensions at large distances. In another branch ($\epsilon = -1$) such cosmic acceleration is not realized.

3

Brane Cosmology

Although brane cosmology is a widely studied topic, it is important to mention that there are no solid models that provide concrete results to the study of the phenomenology of the universe. However, there are important contributions which serve as a framework for the development of new cosmological models. In the previous chapter three of the most important were mentioned, in this chapter we will abound in the primordial results of the Randall-Sundrum models, followed by a formal treatment of the brane cosmology and the brane/bulk based approaches. At the end of the chapter, two-branes configuration is exposed as an introduction to the proposed model of this thesis. Several models study solutions to the dynamic equations of the universe with two branes interacting with each other, and become the basis of this study

3.1 Randall-Sundrum model

The Randall-Sundrum model was conceived in 1999 to address the Higgs Hierarchy Problem in particle physics. It arose enormous interest from theoreticians and phenomenologists and is popular among the builders of extra dimensions theories. Randall Sundrumm models are conceived under the hypothesis that the real world is a higher-dimensional universe described by a warped geometry. More concretely, the visible universe is a five-dimensional anti-de Sitter space and the elementary particles are localized on a $(3 + 1)$-dimensional brane.

3.1.1 RS1 model

In RS1, there are two 3-branes embedded in a five dimensional anti-de Sitter bulk spacetime where x^μ are the familiar four-dimensional coordinates while $0 \leq y \leq y_c$ is the coordinate for the extra dimension. Clearly our space-time cannot fill all of the five dimensions, so it is necessary to specify boundary conditions: identify $(x^\mu, +y)$ with $(x^\mu, -y)$ and take y to be periodic with period $2y_c$. The branes are placed in the orbifold fixed points at $y = 0$ and $y = y_c$ and are taken to have tension λ_0 and λ_c respectively. These fixed points may also be thought of as the boundaries of the five-dimensional spacetime so that the action describing this model is given by

$$S = \frac{1}{2\kappa^2_{(5)}} \int d^4x \int_{-y_c}^{y_c} dy \sqrt{g_5}\,(R_5 - 2\Lambda_5) \; - \lambda_0 \int_{y=0} d^4x \sqrt{g_0} \; - \lambda_c \int_{y=y_c} d^4x \sqrt{g_c} \; . \tag{3.1}$$

where g_5 is the bulk metric and g_0, g_c are the induced metrics on the branes at $y = 0, y_c$ respectively. $2\kappa^2_{(5)} = M_5^{-3}$ is a constant which is related to the five-dimensional Planck mass. In order for the 3-branes to satisfy the four dimensional Poincaré invariance, the metric is choosen to take the following form

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{3.2}$$

The bulk equations of motion with orbifold boundary conditions impose a fine tuning of the brane tensions against the bulk cosmological constant

$$\lambda_0 = -\lambda_c = \frac{6k}{\kappa^2_{(5)}}, \;\; \Lambda_5 = -6k^2 \tag{3.3}$$

This implies $-\frac{\Lambda_5}{6} = \frac{\kappa^4_{(5)}}{36}\lambda_0^2$. We are also free to set $a(0) = 1$ so that we arrive at the following solution for the metric

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \quad \text{for } -y_c \leq y \leq y_c. \tag{3.4}$$

The $\mathbb{Z}_2$ symmetry about $y = 0$ is explicit whereas the other boundary conditions should be understood. We also note that the constant y slicings exhibit Poincaré invariance as required. The metric (3.4) contains an exponential warp factor which is seen graphically in figure 3.2. Notice the peak in the warp factor at the positive tension brane and the trough at the negative tension brane. If

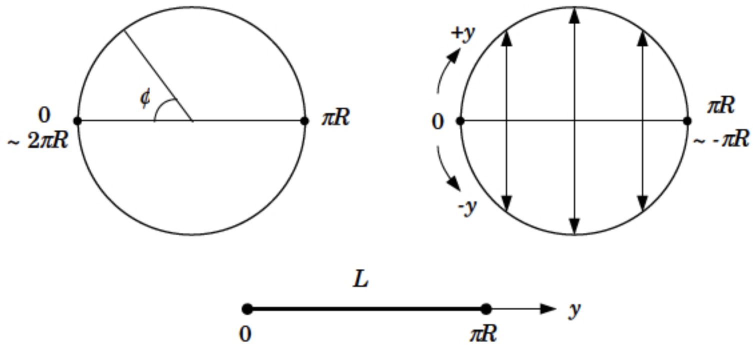

Figure 3.1 Orbifold symmetry. Image taken from `https://www.researchgate.net/publication/239928060_The_Randall-Sundrum_Model`

well the RS1 model is a non realistic model, there are many works which include matter energy distribution or scalar field on the brane, which is the case of this thesis.

3.1.2 Solving the hierarchy problem

In order to tackle the hierarchy problem, we will need to derive the (effective) four-dimensional Planck scale, m_{pl} in terms of the five-dimensional scales M, k, y_c. We do this by identifying the four-dimensional low energy effective theory. This comes from massless graviton fluctuations. In principle, we should also include massless fluctuations in the brane separation, often referred to as the radion field but in this section will assume the brane separation is stabilised at y_c. The gravitational zero modes now take the form

$$ds^2 = e^{-2k|y|}\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu + dy^2 \quad \text{where } \bar{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x) \tag{3.5}$$

and we interpret $h_{\mu\nu}$ as the physical graviton in the four-dimensional effective theory. We now substitute equation (3.5) into the action (3.1) to derive the effective action. Focusing on the curvature term we find that

$$S_{eff} = M^3 \int d^4x \, \sqrt{\bar{g}}\bar{R} \int_{-y_c}^{y_c} dz \; e^{-2k|y|} + \dots \tag{3.6}$$

where $\bar{R}$ is the Ricci scalar built out of $\bar{g}_{\mu\nu}(x)$. We now perform the y-integral to obtain

$$m_{pl}^2 = \frac{M^3}{k}\left[1 - e^{-2ky_c}\right]. \tag{3.7}$$

This tells us that m_{pl} depends weakly on y_c in the limit of large ky_c. We will see that this is not the case for the physical masses in the SM.

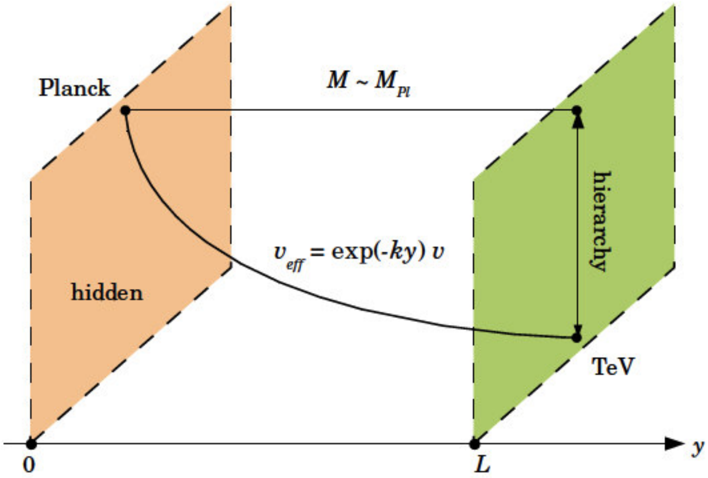

Figure 3.2 Solving hierarchy problem. Image taken from https://www.researchgate.net/publication/239928060_The_Randall-Sundrum_Model

Suppose we live on the negative tension brane at $y = y_c$. Consider a fundamental Higgs field bound to this brane. If it has a five-dimensional mass parameter, m_0, then the matter part of the action near the brane is given by

$$S_c = \int_{y=y_c} d^4x\sqrt{g_c}\left[g_c^{\mu\nu}\nabla_\mu H^\dagger \nabla_\nu H - \lambda\left(|H|^2 - m_0^2\right)^2\right] \tag{3.8}$$

where ∇_μ is the covariant derivative corresponding to g_c. The metric at $y = y_c$ is $\bar{g}_{c\mu\nu} = e^{-2ky_c}\bar{g}_{\mu\nu}$ so that

$$S_c = \int_{y=y_c} d^4x\sqrt{\bar{g}}e^{-4ky_c}\left[e^{2ky_c}\bar{g}^{\mu\nu}\nabla_\mu H^\dagger \nabla_\nu H - \lambda\left(|H|^2 - m_0^2\right)^2\right] \tag{3.9}$$

We now renormalise the Higgs wavefunction, $H \to e^{ky_c}H$, to derive the following part of the effective action

$$S_{eff} = \int_{y=y_c} d^4x\sqrt{\bar{g}}\left[\bar{g}^{\mu\nu}\nabla_\mu H^\dagger \nabla_\nu H - \lambda\left(|H|^2 - e^{-2ky_c}m_0^2\right)^2\right] + \dots \tag{3.10}$$

An observer on the brane will therefore measure the physical mass of the Higgs to be

$$m_H = e^{-ky_c} m_0. \tag{3.11}$$

This result generalises to any mass parameter on the negative tension brane.

We shall now address the hierarchy problem directly. Assume that the bare Higgs mass, m_0, and the fundamental Planck mass, M, are both around 10^{19} GeV, thereby eliminating any hierarchy between the two scales in the five-dimensional theory. The physical masses in the effective theory are given by equations (3.7) and (3.11). To ensure that $m_H \sim 10^3$ GeV and $m_{pl} \sim 10^{19}$ GeV we require that $e^{ky_c} \sim 10^{15}$. The presence of the exponential here is crucial because all we really need is $ky_c \sim 50$. We see that we have solved the hierarchy problem without introducing a second hierarchy involving the compactification scale, $1/y_c$ or the AdS length, $1/k$. We should emphasize here that this is only true if the radion is stabilised. If not, its fluctuations appear in the exponential, spoiling the solution to the problem.

3.2 Brane dynamics

Randall-Sundrum braneworlds provide a radical new way of thinking about our universe and the extra dimensions that might exist. If this extra dimension is warped anti-de Sitter space then it can be infinitely large and still exhibit localisation of gravity on the brane. A more general treatment is present in this section in which the basic idea is to use the Gauss-Codazzi formalism to project the 5D curvature along the brane.

The 5D field equations determine the 5D curvature tensor; in the bulk, they are

$${}^{(5)}G_{AB} = -\Lambda_5 \, {}^{(5)}g_{AB} + \kappa_5^2 \, {}^{(5)}T_{AB}, \tag{3.12}$$

where ${}^{(5)}T_{AB}$ represents any 5D energy-momentum of the bulk, as a 5D scalar field.

Let y be a Gaussian normal coordinate orthogonal to the brane (which is placed at $y = 0$), so that $n_A dX^A = dy$, with n^A being the unit normal. The 5D metric in terms of the induced metric on $\{y = \text{const.}\}$ surfaces is locally given by

$${}^{(5)}g_{AB} = g_{AB} + n_A n_B, \qquad {}^{(5)}ds^2 = g_{\mu\nu}(x^\alpha, y) dx^\mu dx^\nu + dy^2. \tag{3.13}$$

The extrinsic curvature of $\{y = \text{const.}\}$ surfaces describes the embedding of these surfaces. It can be defined via the Lie derivative or via the covariant derivative:

$$K_{AB} = \frac{1}{2}\pounds_{\mathbf{n}}\, g_{AB} = g_A{}^C\, {}^{(5)}\nabla_C n_B, \tag{3.14}$$

so that

$$K_{[AB]} = 0 = K_{AB}n^B, \tag{3.15}$$

where square brackets denote anti-symmetrization. The Gauss equation gives the 4D curvature tensor in terms of the projection of the 5D curvature, with extrinsic curvature corrections:

$$R_{ABCD} = {}^{(5)}R_{EFGH}\, g_A{}^E g_B{}^F g_C{}^G g_D{}^H + 2K_{A[C}K_{D]B}, \tag{3.16}$$

and the Codazzi equation determines the change of K_{AB} along $\{y = \text{const.}\}$ via

$$\nabla_B K^B{}_A - \nabla_A K = {}^{(5)}R_{BC}\, g_A{}^B n^C, \tag{3.17}$$

where $K = K^A{}_A$.

Some other useful projections of the 5D curvature are:

$$\begin{aligned} {}^{(5)}R_{EFGH}\, g_A{}^E g_B{}^F g_C{}^G n^H &= 2\nabla_{[A}K_{B]C}, && (3.18)\\ {}^{(5)}R_{EFGH}\, g_A{}^E n^F g_B{}^G n^H &= -\pounds_{\mathbf{n}}K_{AB} + K_{AC}K^C{}_B, && (3.19)\\ {}^{(5)}R_{CD}\, g_A{}^C g_B{}^D &= R_{AB} - \pounds_{\mathbf{n}}K_{AB} - KK_{AB} + 2K_{AC}K^C{}_B. && (3.20) \end{aligned}$$

The 5D curvature tensor has Weyl (tracefree) and Ricci parts:

$${}^{(5)}R_{ABCD} = {}^{(5)}C_{ACBD} + \frac{2}{3}\left({}^{(5)}g_{A[C}\, {}^{(5)}R_{D]B} - {}^{(5)}g_{B[C}\, {}^{(5)}R_{D]A}\right) - \frac{1}{6}{}^{(5)}g_{A[C}\, {}^{(5)}g_{D]B}\, {}^{(5)}R. \tag{3.21}$$

3.2.1 Field equations on the brane

Using Equations (3.12) and (3.16), it follows that

$$\begin{aligned} G_{\mu\nu} &= -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \frac{2}{3}\kappa_5^2\left[{}^{(5)}T_{AB}g_\mu{}^A g_\nu{}^B + \left({}^{(5)}T_{AB}n^A n^B - \frac{1}{4}\,{}^{(5)}T\right)g_{\mu\nu}\right]\\ &\quad + KK_{\mu\nu} - K_\mu{}^\alpha K_{\alpha\nu} + \frac{1}{2}\left[K^{\alpha\beta}K_{\alpha\beta} - K^2\right]g_{\mu\nu} - \mathcal{E}_{\mu\nu}, \end{aligned} \tag{3.22}$$

where ${}^{(5)}T = {}^{(5)}T^A{}_A$, and where

$$\mathcal{E}_{\mu\nu} = {}^{(5)}C_{ACBD}\, n^C n^D g_\mu{}^A g_\nu{}^B, \tag{3.23}$$

is the projection of the bulk Weyl tensor orthogonal to n^A. This tensor satisfies

$$\mathcal{E}_{AB} n^B = 0 = \mathcal{E}_{[AB]} = \mathcal{E}_A{}^A, \tag{3.24}$$

by virtue of the Weyl tensor symmetries. Evaluating Equation (3.22) on the brane (strictly, as $y \to \pm 0$, since $\mathcal{E}_{AB}$ is not defined on the brane) will give the field equations on the brane.

First, we need to determine $K_{\mu\nu}$ at the brane from the junction conditions. The total energy-momentum tensor on the brane is

$$T^{\rm brane}_{\mu\nu} = T_{\mu\nu} - \lambda g_{\mu\nu}, \tag{3.25}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of particles and fields confined to the brane (so that $T_{AB} n^B = 0$). The 5D field equations, including explicitly the contribution of the brane, are then

$${}^{(5)}G_{AB} = -\Lambda_5\, {}^{(5)}g_{AB} + \kappa_5^2 \left[{}^{(5)}T_{AB} + T^{\rm brane}_{AB} \delta(y) \right]. \tag{3.26}$$

Here the delta function enforces in the classical theory the string theory idea that Standard Model fields are confined to the brane. This is not a gravitational confinement, since there is in general a nonzero acceleration of particles normal to the brane.

Integrating Equation (3.26) along the extra dimension from $y = -\epsilon$ to $y = +\epsilon$, and taking the limit $\epsilon \to 0$, leads to the Israel–Darmois junction conditions at the brane,

$$g^+_{\mu\nu} - g^-_{\mu\nu} = 0, \tag{3.27}$$

$$K^+_{\mu\nu} - K^-_{\mu\nu} = -\kappa_5^2 \left[T^{\rm brane}_{\mu\nu} - \frac{1}{3} T^{\rm brane} g_{\mu\nu} \right], \tag{3.28}$$

where $T^{\rm brane} = g^{\mu\nu} T^{\rm brane}_{\mu\nu}$. The Z_2 symmetry means that when you approach the brane from one side and go through it, you emerge into a bulk that looks the same, but with the normal reversed, $n^A \to -n^A$. Then Equation (3.14) implies that

$$K^-_{\mu\nu} = -K^+_{\mu\nu}, \tag{3.29}$$

so that we can use the junction condition Equation (3.28) to determine the extrinsic curvature on the brane:

$$K_{\mu\nu} = -\frac{1}{2}\kappa_5^2 \left[T_{\mu\nu} + \frac{1}{3}(\lambda - T)\, g_{\mu\nu} \right], \tag{3.30}$$

where $T = T^{\mu}{}_{\mu}$, where we have dropped the $(+)$, and where we evaluate quantities on the brane by taking the limit $y \to +0$.

Finally we arrive at the induced field equations on the brane, by substituting Equation (3.30) into Equation (3.22):

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + 6\frac{\kappa^2}{\lambda}\mathcal{S}_{\mu\nu} - \mathcal{E}_{\mu\nu} + 4\frac{\kappa^2}{\lambda}\mathcal{F}_{\mu\nu}. \tag{3.31}$$

The 4D gravitational constant is an effective coupling constant inherited from the fundamental coupling constant, and the 4D cosmological constant is nonzero when the RS balance between the bulk cosmological constant and the brane tension is broken:

$$\kappa^2 \equiv \kappa_4^2 = \frac{1}{6}\lambda\kappa_5^4, \tag{3.32}$$

$$\Lambda = \frac{1}{2}\left[\Lambda_5 + \kappa^2\lambda\right]. \tag{3.33}$$

The first correction term relative to Einstein's theory is quadratic in the energy-momentum tensor, arising from the extrinsic curvature terms in the projected Einstein tensor:

$$\mathcal{S}_{\mu\nu} = \frac{1}{12}TT_{\mu\nu} - \frac{1}{4}T_{\mu\alpha}T^{\alpha}{}_{\nu} + \frac{1}{24}g_{\mu\nu}\left[3T_{\alpha\beta}T^{\alpha\beta} - T^2\right]. \tag{3.34}$$

The second correction term is the projected Weyl term. The last correction term on the right of Equation (3.31), is

$$\mathcal{F}_{\mu\nu} = {}^{(5)}T_{AB}\, g_{\mu}{}^{A} g_{\nu}{}^{B} + \left[{}^{(5)}T_{AB}n^A n^B - \frac{1}{4}\,{}^{(5)}T\right] g_{\mu\nu}, \tag{3.35}$$

where ${}^{(5)}T_{AB}$ describes any stresses in the bulk apart from the cosmological constant.

3.2.2 Modified Friedmann equations

To obtain the equivalent Friedmann equations in brane worlds, it is instructive to use Gaussian normal coordinates, in which the brane is fixed but the bulk metric is not static at all.

$$ {}^{(5)}ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] + dy^2. \tag{3.36} $$

Here $a_0 = a_0(t) = a(t,0)$ is the scale factor on the FRW brane at $y = 0$, and t may be chosen as proper time on the brane, so that $n_0 = n(t,0) = 1$. Then the metric functions are

$$ n = \frac{\dot{a}}{\dot{a}_0}, \tag{3.37} $$

$$ a = a_0\left[\cosh\left(\frac{y}{\ell}\right) - \left\{1 + \frac{\rho(t)}{\lambda}\right\}\sinh\left(\frac{|y|}{\ell}\right)\right]. \tag{3.38} $$

The junction conditions determine the Friedmann equation. The extrinsic curvature at the brane is

$$ K^\mu{}_\nu = diag\left(\frac{n'}{n}, \frac{a'}{a}, \frac{a'}{a}, \frac{a'}{a}\right)_{\rm y=0} \tag{3.39} $$

Then, by Equation (3.30),

$$ \left.\frac{n'}{n}\right|_0 = \frac{\kappa_5^2}{6}(2\rho + 3p - \lambda), \tag{3.40} $$

$$ \left.\frac{a'}{a}\right|_0 = -\frac{\kappa_5^2}{6}(\rho + \lambda). \tag{3.41} $$

The field equations yield the first integral

$$ (aa')^2 - \left(\frac{a\dot{a}}{n}\right)^2 + \frac{\Lambda_5}{6}a^4 - ka^2 + C_{DR} = 0, \tag{3.42} $$

where C_{DR} is constant corresponding with the dark radiation contribution. Evaluating this at the brane, using Equation (3.41), gives the modified Friedmann equations

$$ H_0^2 = \frac{\kappa^2}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{C}{a_0^4} + \frac{1}{3}\Lambda - \frac{K}{a_0^2}, \tag{3.43} $$

Differentiating Equation (3.43) and using the energy conservation equation, we obtain

$$ \dot{H}_0 = -\frac{\kappa^2}{2}(\rho + p)\left(1 + \frac{\rho}{\lambda}\right) - 2\frac{C}{a_0^4} + \frac{K}{a_0^2}, \tag{3.44} $$

In order to recover the observational successes of general relativity, the high-energy regime where significant deviations occur must take place before nucleosynthesis, i.e., cosmological observations impose the lower limit

$$\lambda > (1\ \mathrm{MeV})^4 \quad \Rightarrow \quad M_5 > 10^4\ \mathrm{GeV}. \tag{3.45}$$

3.2.3 Inflation on the brane

Now it is instructive that the field satisfies the Klein–Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{3.46}$$

In 4D general relativity, the condition for inflation, $\ddot{a} > 0$, is $\dot{\phi}^2 < V(\phi)$, i.e., $p < -\frac{1}{3}\rho$, where $\rho = \frac{1}{2}\dot{\phi}^2 + V$ and $p = \frac{1}{2}\dot{\phi}^2 - V$. The modified Friedmann equation leads to a stronger condition for inflation: Using Equation (3.43), with $m = 0 = \Lambda = K$, and Equation (3.46), we find that

$$\ddot{a} > 0 \quad \Rightarrow \quad w < -\frac{1}{3}\left[\frac{1 + 2\rho/\lambda}{1 + \rho/\lambda}\right], \tag{3.47}$$

where the square brackets enclose the brane correction to the general relativity result. As $\rho/\lambda \to 0$, the 4D result $w < -\frac{1}{3}$ is recovered, but for $\rho > \lambda$, w must be more negative for inflation. In the very high-energy limit $\rho/\lambda \to \infty$, we have $w < -\frac{2}{3}$. When the only matter in the universe is a self-interacting scalar field, the condition for inflation becomes

$$\dot{\phi}^2 - V + \left[\frac{\frac{1}{2}\dot{\phi}^2 + V}{\lambda}\left(\frac{5}{4}\dot{\phi}^2 - \frac{1}{2}V\right)\right] < 0, \tag{3.48}$$

which reduces to $\dot{\phi}^2 < V$ when $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V \ll \lambda$.

In the slow-roll approximation, we get

$$H^2 \approx \frac{\kappa^2}{3} V \left[1 + \frac{V}{2\lambda}\right], \tag{3.49}$$

$$\dot{\phi} \approx -\frac{V'}{3H}. \tag{3.50}$$

The brane-world correction term V/λ in Equation (3.49) serves to enhance the Hubble rate for a given potential energy, relative to general relativity. Thus there is enhanced Hubble 'friction' in Equation (3.50), and brane-world effects will reinforce slow-roll at the same potential energy. We can see this by defining slow-roll parameters that reduce to the standard parameters in the low-energy limit:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_{\rm p}^2}{16\pi}\left(\frac{V'}{V}\right)^2\left[\frac{1+V/\lambda}{(1+V/2\lambda)^2}\right], \tag{3.51}$$

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{M_{\rm p}^2}{8\pi}\left(\frac{V''}{V}\right)\left[\frac{1}{1+V/2\lambda}\right]. \tag{3.52}$$

Self-consistency of the slow-roll approximation then requires $\epsilon, |\eta| \ll 1$. At low energies, $V \ll \lambda$, the slow-roll parameters reduce to the standard form. However at high energies, $V \gg \lambda$, the extra contribution to the Hubble expansion helps damp the rolling of the scalar field, and the new factors in square brackets become $\approx \lambda/V$:

$$\epsilon \approx \epsilon_{\rm gr}\left[\frac{4\lambda}{V}\right], \qquad \eta \approx \eta_{\rm gr}\left[\frac{2\lambda}{V}\right], \tag{3.53}$$

where $\epsilon_{\rm gr}, \eta_{\rm gr}$ are the standard general relativity slow-roll parameters.

The number of e-folds during inflation, $N = \int H dt$, is, in the slow-roll approximation,

$$N \approx -\frac{8\pi}{M_{\rm p}^2}\int_{\phi_{\rm i}}^{\phi_{\rm f}} \frac{V}{V'}\left[1+\frac{V}{2\lambda}\right] d\phi. \tag{3.54}$$

Brane-world effects at high energies increase the Hubble rate by a factor $V/2\lambda$, yielding more inflation between any two values of ϕ for a given potential. Thus we can obtain a given number of e-folds for a smaller initial inflaton value $\phi_{\rm i}$. For $V \gg \lambda$, Equation (3.54) becomes

$$N \approx -\frac{128\pi^3}{3M_5^6}\int_{\phi_{\rm i}}^{\phi_{\rm f}} \frac{V^2}{V'} d\phi. \tag{3.55}$$

3.3 Two branes in a five dimensional bulk

In this thesis we consider that space is 5-dimensional and contains two four-dimensional branes, which are spatially homogeneous, isotropic and time-independent and located at $y = 0$ and $y = y_c$. The fifth dimension is periodic and has a reflection symmetry of (orbifold) with respect to each of the branes. Therefore, the space between the two membranes is only half of the space along the fifth dimension. This model is the simplest and we will only limit ourselves to calculating some relationships between the branes. In this section the formalism of the following authors is handled [13], [88], [14], [44].

3.3.1 Topological constraints

The primary result of this thesis is based on the hypothesis that when considering two branes, there is necessarily an interdependence between their properties and, as a consequence, some phenomena observed on one brane are caused by some phenomenon on the other brane. These restrictions are the product of the modification of the geometry near each of the branes.

Following the treatment made by Binetruy et. al. [13], is possible, starting form the metric

$$ds^2 = -n^2 dt^2 + a^2 g_{\mu\nu} dx^\mu dx^\nu + b^2 dy^2, \tag{3.56}$$

to solve the Einstein's equations, from the global point of view, leading to the second derivative of scalar factor a necessarily takes the form

$$a'' = [a']_0 \left(\delta(y) - \delta(y - y_c)\right) + \left([a']_0 + [a']_c\right) \left(\delta(y - y_c) - 1\right), \tag{3.57}$$

where $[a']_0$ and $[a']_c$, are the jump of a' on the first and second branes and they are equivalent to the energy distribution in each brane:

$$\begin{aligned} \frac{[a']_0}{a_0 b_0} &= -\frac{\kappa 5^2}{3}\rho_0, && (3.58)\\ \frac{[a']_c}{a_c b_c} &= -\frac{\kappa 5^2}{3}\rho_c. && (3.59) \end{aligned}$$

Integrating (3.57) over y yields the following solution for a:

$$a = a_0 + \left(1/2|y| - 1/2y^2\right)[a']_0 - 1/2y^2[a']_c, \tag{3.60}$$

A similar expression is obtained for n. Allowing a linear dependence in y for the function b, we write:

$$b = b_0 + 2|y|(b_c - b_0), \tag{3.61}$$

where b_0 is assumed to be constant in time. These two solutions for a and b metric coefficients are used to obtain (with a little algebra) the equations

$$\begin{aligned} \rho_0 a_0 &= -\rho_c a_{1/2}, && (3.62)\\ (2\rho_0 + 3p_0) n_0 &= -(2\rho_c + 3p_c) n_c. && (3.63) \end{aligned}$$

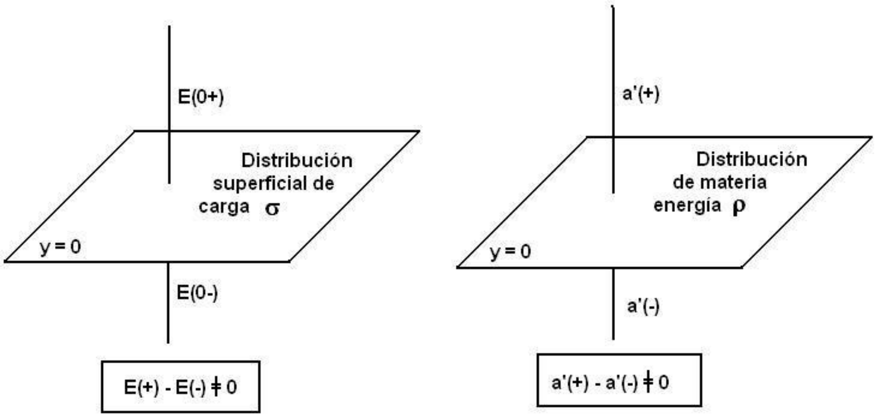

Figure 3.3 Analogy with electrostatic

Hence the matter on one brane is constrained by the matter on the other. Authors in [14] suggest the constraints between the two branes obtained above can probably be seen as a particular example of "topological constraints", which impose specific restrictions on the distribution of localized matter in a space that contains compact dimensions and which can be found in many different contexts (D-branes, orientifolds or topological defects).

Analyzing the simplest solutions, one can envisage linear solutions in $|y|$ for a and n, of the form

$$a = a_0(t)\,(1+\lambda|y|), \tag{3.64}$$

$$n = n_0(t)\,(1+\mu|y|), \tag{3.65}$$

$$b = b_0, \tag{3.66}$$

where b_0 is assumed to be constant in time. λ and μ are in general functions of time and depend directly on the matter content of the brane. They are obtained from boundary conditions,

$$\lambda = -\frac{\kappa_{(5)}^2}{6} b_0 \rho_0, \qquad \mu = \frac{\kappa_{(5)}^2}{2}\left(\omega_0 + \frac{2}{3}\right) b_0 \rho_0. \tag{3.67}$$

We have introduced $\omega_0 \equiv p_0/\rho_0$ which is not necessarily a constant. Note that the metric is well defined for $\kappa_{(5)}^2 b_0 \rho < 1$.

An interesting consequence of our solution is the behaviour of the second brane. As in the previous section, the matter content of the second brane is totally determined by the "topological constraints" due to the compactness of the additional dimension. Since our solution is a particular case of the general form (3.60), the same relations (3.62) and (3.63) apply, which implies

$$\rho_c = -\rho_0 \left(1 - \frac{\kappa_{(5)}^2}{6} b_0 \rho_0 y_c\right)^{-1}, \tag{3.68}$$

and

$$2 + 3w_c = (2 + 3\omega_0)\, \frac{1 - \frac{\kappa_{(5)}^2}{6} b_0 \rho_0 y_c}{1 + (2 + 3\omega_0)\frac{\kappa_{(5)}^2}{6} b_0 \rho_0 y_c}, \tag{3.69}$$

where $w_c \equiv p_c/\rho_c$. In the general case, w_0 and w_c will be time-dependent. Even if w_0 is chosen to be constant, w_c will be time-dependent, except in two particular cases: $w_0 = w_c = -1$ corresponding to a cosmological constant on both branes (although of opposite signs), and $w_0 = w_c = -2/3$.

4

The model

In the context of two-brane models with matter fields, one can naturally to ask if the parameters which determine the evolution in time in both branes are related. This is the central idea of present thesis. Binetruy et al. [14] have shown that there exists an equation which relate the fields in both branes assuming a mutual interaction between them, through a topological constrains. For example, in [28] the authors assume that the hidden brane is dominated by a scalar field, trying to reproduce the dark matter effects in the visible brane. Based in the RS models and in the previous results founded by [13], this thesis work focuses in generalize the solution showed in the last section for a vacuum 5D bulk, in which we propose the metric coefficients have a particular mathematical structure. This formalism generates a dynamical equation for the Hubble parameter in hidden brane H_c closely related with the dynamics of the visible brane H_0; in the other words, the fields immersed in one brane generates dynamics in the other brane through gravitational effects. First, we study a toy model with equations of state (EoS) constant and the repercussions in the mutual brane evolution; on the other hand we study as a particular field election a scalar field as responsible of the inflationary dynamics in the hidden brane.

4.1 Cosmology for a two branes system

The model for this thesis, consist of a two-brane system embedded in a 5dim manifold, in which the fifth extra dimension is represented by the coordinate y, and then the branes will be located at $y = 0$ (which will represent our visible universe), and at $y = y_c$ (which will be called the hidden universe), respectively. I write the action for this system as

$$S = -\frac{1}{2\kappa^2_{(5)}} \int d^5x \sqrt{-g_{(5)}} R_{(5)} + \int d^4x \sqrt{-g_{(4)}} (L_0 + L_c), \tag{4.1}$$

where $g_{(5)}$, $\kappa_{(5)}$ and $R_{(5)}$ are the 5-dimensional metric, gravitational constant and the Ricci scalar respectively. The most general metric for this model can be written in the form

$$ds^2 = -n^2(t, |y|)dt^2 + a^2(t, |y|)g_{ij}dx^i dx^j + b^2(t, |y|)dy^2\,, \tag{4.2}$$

where $n(t, |y|)$, $a(t, |y|)$ and $b(t, |y|)$ are general functions, whereas g_{ij} is the 3dim metric. As an important feature, we impose the symmetries enumerated in the following way:

1. Reflection, $(x^\mu, y) \to (x^\mu, -y)$
2. Compactification, $(x^\mu, y) \to (x^\mu, y + 2iy_c)$, $i = 1, 2, \ldots$

Similarly, we demand that each metric coefficients $a(t, |y|)$, $n(t, |y|)$ and $b(t, |y|)$ are subjected to the conditions [88]

$$[F']_0 = 2F'|_{y=0+}, \tag{4.3}$$

$$[F']_c = -2F'|_{y=y_c-}, \tag{4.4}$$

$$F'' = \frac{d^2F(t, |y|)}{d\,|y|^2} + [F']_0\,\delta(y) + [F']_c\,\delta(y - y_c), \tag{4.5}$$

where the prime denotes derivate with respect to y, the square brackets denotes the discontinuity in the first derivative at the positions $y = 0$ and $y = y_c$ and F is a generic function which meets the above conditions [88].

The equation 4.5 is obtained if we demand that $d\,|y|\,/dy = 1$, and $d^2\,|y|\,/dy^2 = 2\delta(y) - 2\delta(y - y_c)$, for $y \in [0, y_c]$. The subindex 0 will be used for quantities valued at $y = 0$, whereas a subindex c will be used for quantities valued at $y = y_c$.

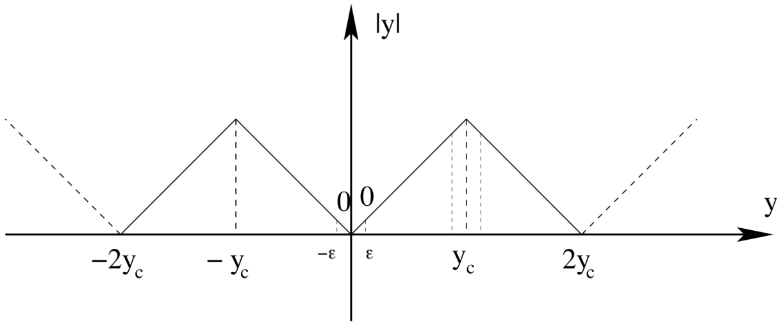

Figure 4.1 The function $|y|$ defined in a cyclic coordinate y

Notice that from here, we use units in which $c = \hbar = 1$, and that the 5dim metric has the surplus signature diag$(-,+,+,+,+)$. We now consider the validity of Einstein's equations in five dimensions, $\tilde{G}_{AB} = \kappa^2_{(5)}\tilde{T}_{AB}$,

Also, we will assume that the two branes are dominated by perfect fluid matter components that satisfy the following barotropic equations of state (EoS): $p_0 = \omega_0\rho_0$, and $p_c = \omega_c\rho_c$, respectively. Here, p_0, p_c are the pressure, ρ_0, ρ_c are the energy density and ω_0, ω_c are the EoS of the visible and hidden brane, respectively. Then, the energy-momentum tensor is written as

$$\tilde{T}^A_B = -\frac{\Lambda_5}{\kappa^2_{(5)}}g_{AB} + \frac{\delta(y)}{b_0}\text{diag}(-\rho_0, \mathbf{p}_0, 0) + \frac{\delta(y-y_0)}{b_c}\text{diag}(-\rho_c, \mathbf{p}_c, 0)\,, \tag{4.6}$$

where the first term corresponds to the bulk contribution of a 5dim cosmological constant (CC) Λ_5, $\kappa^4_{(5)} \equiv 6\kappa^2_{(4)}/\lambda_0$, where λ_0 is the brane tension and $\kappa_{(4)}$ is the 4dim gravitational constant; finally, the second and third term correspond to the matter fields contained in the two branes. Taking the conservation of energy-momentum tensor, $\bigtriangledown_A\tilde{T}^A_B = 0$ immediately yields the known results for each brane

$$\dot{\rho}_0 + 3(p_0+\rho_0)\frac{\dot{a}_0}{a_0} = 0 \tag{4.7}$$

where a dot denotes derivative with respect to time. We will further assume the perfect fluid components satisfy equations of state in the form $p_0 = \omega_0\rho_0$, and

$p_c = \omega_c \rho_c$. Up to this point, both equations of state are free functions of time, but some particular cases will be discussed in further chapters.

The matter fields and the metric functions are constrained by the conditions at $y = 0$ and $y = y_c$[14, 88]

$$\frac{a'|_0}{a_0 b_0} = -\frac{\kappa^2_{(5)}}{6}\rho_0\,, \quad \frac{a'|_c}{a_c b_c} = +\frac{\kappa^2_{(5)}}{6}\rho_c\,, \tag{4.8}$$

$$\frac{n'|_0}{n_0 b_0} = -\frac{\kappa^2_{(5)}}{3}(3p_0 + 2\rho_0)\,, \quad \frac{n'|_c}{n_c b_c} = +\frac{\kappa^2_{(5)}}{3}(3p_c + 2\rho_c)\,, \tag{4.9}$$

4.2 First generalization and master thesis work

In a previous work [69], which is the subject of the Master thesis degree, we have showed that it is possible generalize the metric presented by [76] by choosing a particular form of the metric coefficients a and b. Let us begin with an ansatz for the metric coefficient $a(t, |y|)$ such that it depends on two time dependent parameters, $\lambda(t)$ and $\alpha(t)$, as

$$a(t, |y|) = \alpha f(\lambda\,|y|)\,. \tag{4.10}$$

Function f, and its derivative f', are well defined at $[0, y_c]$, actually $f(0) = 1$. In this way,

$$a(t, |y|) = a_0 f(\lambda\,|y|)\,. \tag{4.11}$$

According to Eq. (4.4) , we obtain

$$[a']_0 = 2\lambda a_0 f'_0, \quad [a']_c = -2\lambda a_0 f'_c\,, \tag{4.12}$$

where $f' \equiv \frac{df(\lambda|y|)}{d(\lambda|y|)}$. Note, λ is a term wich is not variable in the y-coordinate; so, in the boundary of the two branes, its value must be proportional to energy density in each brane. This implies energy density components in both branes are not evolving as separated entities, but are related. Using Eqs. (4.8), in (4.12), we find

$$\kappa^2_{(5)}\rho_c = -\frac{b_0 f'_c}{b_c f'_0 f_c}\kappa^2_{(5)}\rho_0. \tag{4.13}$$

In the last equation, f_c and f'_c are functions of λ, and consequently, functions of ρ_0, namely,

$$\lambda = -\frac{b_0}{6 f'_0}\kappa^2_{(5)}\rho_0\,. \tag{4.14}$$

These last two results show that there exists a connection between the two branes just in the form of topological constraints. Now, we are interested in the connection between the equations of state in each brane. In analogy to Eq. (4.13), but now for the ansatz

$$n(t, |y|) = g(\beta\,|y|), \tag{4.15}$$

where $g(0) = 1$, and using Eqs. (4.4) and (4.9), , we find

$$\kappa_{(5)}^2 \rho_c (2 + 3\omega_c) = -\frac{b_0 g_c'}{b_c g_0' g_c} \kappa_{(5)}^2 \rho_0 (2 + 3\omega_0)\,, \tag{4.16}$$

where $g' \equiv \frac{dg(\beta|y|)}{d(\beta|y|)}$. In the former equation, both g_c and g_c' are functions of β, where

$$\beta = \frac{b_0}{6g_0'} \kappa_{(5)}^2 \rho_0 (2 + 3\omega_0)\,. \tag{4.17}$$

Combining Eq. (4.16) with Eq. (4.13), we find that ω_c is connected to ω_0 and ρ_0, namely,

$$(2 + 3\omega_c) = \frac{f_c}{f_c'} \frac{g_c'}{g_c} \frac{f_0'}{g_0'} (2 + 3\omega_0))\,. \tag{4.18}$$

This last equation gives us the relationship between the equations of state valued at the position of each brane. Eqs. (4.13), and (4.18), are the generalization of (3.68) and (3.69), and both are the main results of this section. Some of the typical $f(\lambda\,|y|)$ and $g(\beta\,|y|)$ are showed in Table (4.1)

$f(\lambda\,\lvert y\rvert)$	$g(\beta\,\lvert y\rvert)$	λ	β	ρ_c/ρ_0	$\frac{2+3\omega_c}{2+3\omega_0}$
$1+\lambda\,\lvert y\rvert$	$1+\beta\,\lvert y\rvert$	$-b_0\kappa_{(5)}^2\rho_0/6$	$b_0\kappa_{(5)}^2\rho_0(2+3\omega_0)/6$	$-\frac{b_0}{b_c(1+\lambda y_c)}$	$\frac{1+\lambda y_c}{1+\beta y_c}$
$(1+\lambda\,\lvert y\rvert)^l$	$(1+\beta\,\lvert y\rvert)^m$	$-b_0\kappa_{(5)}^2\rho_0/6l$	$b_0\kappa_{(5)}^2\rho_0(2+3\omega_0)/6m$	$-\frac{b_0}{b_c(1+\lambda y_c)}$	$\frac{1+\lambda y_c}{1+\beta y_c}$
$e^{\lambda\lvert y\rvert}$	$e^{\beta\lvert y\rvert}$	$-b_0\kappa_{(5)}^2\rho_0/6$	$b_0\kappa_{(5)}^2\rho_0(2+3\omega_0)/6$	$-\frac{b_0}{b_c}$	1

Table 4.1 Analysis for three types of functions f and g: linear, potential and exponential

As an example, let us take $b = 1$, and

$$a(t, |y|) = a_0(t) e^{\lambda|y|}\,, \quad n(t, |y|) = e^{\beta|y|}\,. \tag{4.19}$$

We can see that $f_0' = g_0' = 1$, $f_c = f_c'$ and $g_c = g_c'$, and therefore $\omega_0 = \omega_c$. When $\lambda = \beta < 0$, which corresponds to Anti de Sitter bulk, we have $\omega_0 = -1$ and it is precisely the case of the RS setup [76]. Two instructive works were published analyzing some of the possible phenomenology in this configuration.

In this section the form of the metric coefficients has been restricted to take the functional forms (4.10) y (4.15) Inspired in these works, we will develop a more general treatment (second generalization) by solving the Einstein equations with a particular ansatz and then will include a scalar field as a source of inflation.

4.3 5-D Einstein equations

Before to arrive to the dynamical analysis of the brane systems, we want to establish a correct set of solutions to the Einstein Equations, which are presented as a coupled system of non-homogeneous second-order partial differential equations.

The five-dimensional non zero Einstein tensors, $\tilde{G}_{AB}$, for the metric (4.2) are written as a non-linear differential equations system

$$
\begin{aligned}
\tilde{G}_{00} &= 3\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)-3\frac{n^2}{b^2}\left[\frac{a''}{a}+\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)\right]+3k\frac{n^2}{a^2},\\
\tilde{G}_{ij} &= \frac{a^2}{b^2}\delta_{ij}\left\{\frac{a'}{a}\left(\frac{a'}{a}+2\frac{n'}{n}\right)-\frac{b'}{b}\left(\frac{n'}{n}+2\frac{a'}{a}\right)\right\}\\
&+ \frac{a^2}{b^2}\delta_{ij}\left\{2\frac{a''}{a}+\frac{n''}{n}\right\}+\frac{a^2}{n^2}\delta_{ij}\left\{\frac{\dot{a}}{a}\left(-\frac{\dot{a}}{a}+2\frac{\dot{n}}{n}\right)\right\}\\
&+ \frac{a^2}{n^2}\delta_{ij}\left\{-2\frac{\ddot{a}}{a}+\frac{\dot{b}}{b}\left(-2\frac{\dot{a}}{a}+\frac{\dot{n}}{n}\right)-\frac{\ddot{b}}{b}\right\}-k\delta_{ij},\\
\tilde{G}_{05} &= 3\left(\frac{\dot{a}}{a}\frac{n'}{n}+\frac{\dot{b}}{b}\frac{a'}{a}-\frac{\dot{a}'}{a}\right),\\
\tilde{G}_{55} &= 3\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right)-3\frac{b^2}{n^2}\left[\frac{\ddot{a}}{a}+\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}-\frac{\dot{n}}{n}\right)\right]-3k\frac{b^2}{a^2}.
\end{aligned}
$$

Before to continue, it is necessary to rewrite the above equations in a more convenient way by using the new variables,

$$
X=\frac{a\dot{a}}{n},\quad Y=\frac{aa'}{b},\quad Z=\frac{(an)'}{b},\quad W=\frac{\dot{(ab)}}{n}. \tag{4.20}
$$

In the interval $0<y<y_c$, where Einstein's equatios are valid, $\tilde{G}_{AB}=\kappa^2_{(5)}\tilde{T}_{AB}$, reads

$$
\begin{aligned}
\frac{Y'}{b} - \frac{XW}{ab} &= k - \frac{1}{3}a^2\Lambda_5, && (4.21)\\
\frac{Y'}{b} - \frac{\dot{X}}{n} + \frac{a}{bn}(Z' - \dot{W}) &= k - a^2\Lambda_5, && (4.22)\\
W &= a\frac{X'}{Y}, && (4.23)\\
Z &= a\frac{\dot{Y}}{X}, && (4.24)\\
\frac{\dot{X}}{n} - \frac{YZ}{an} &= -k + \frac{1}{3}a^2\Lambda_5, && (4.25)
\end{aligned}
$$

Substituting (4.23) in (4.21) and integrating, we obtain

$$
Y^2 - X^2 = a^2 k - a^4\frac{\Lambda_5}{6} + constant, \tag{4.26}
$$

which is the generalization of Eq. (3.42), if we identify the constant with C_{DR}. Note that the Eq. (4.23) and Eq. (4.24) are both the same and can be reduced to

$$
\frac{\dot{b}}{b} = \frac{n}{a'}\left(\frac{X}{a}\right)' \tag{4.27}
$$

Last set of equations is solvable only when X is an explicitly function of a

4.3.1 General solutions with separation of variables

We can solve (4.26) and (4.27) exactly when the metric coefficients can be written as

$$
\begin{aligned}
a &= a_t(t)a_y(y) && (4.28)\\
n &= n(y) && (4.29)\\
b &= b(t) && (4.30)
\end{aligned}
$$

Under this ansatz and using the boundary conditions (4.8) and (4.9), Eq. (4.27) give the solutions

$$
\begin{aligned}
n &= k_1 a_y^{\alpha} && (4.31)\\
b &= k_2 a_t^{1-\alpha} && (4.32)\\
\alpha &= -(2 + 3\omega_0) = -(2 + 3\omega_c) && (4.33)
\end{aligned}
$$

where $k_1,\ k_2,\ \alpha$ are all constants. Note in this case, $\omega_0 = \omega_c$ both constants, and therefore, the evolution is similar in both branes. Using this results in (4.26), the metric coefficient $a(t, y)$ must satisfy the follow differential equation.

$$\begin{aligned} \left(\frac{a_y'}{k_2 a_y}\right)^2 a_t^{2(\alpha-1)} - \left(\frac{\dot{a}_t}{k_1 a_t}\right)^2 a_y^{-2\alpha} = \\ = \ k a_t^{-2} a_y^{-2} - \frac{\Lambda_5}{6} + C_{DR} a_t^{-4} a_y^{-4}, \end{aligned} \tag{4.34}$$

This last equation is solvable only in four cases:

1. $k \neq 0; \quad \Lambda_5 = C_{DR} = 0; \quad \alpha = 0, 1$
2. $\Lambda_5 \neq 0; \quad K = C_{DR} = 0; \quad \alpha = 0, 1$
3. $C_{DR} \neq 0; \quad K = \Lambda_5 = 0; \quad \alpha = 3, -3$
4. $k = \Lambda_5 = C_{DR} = 0; \quad \alpha = any$

The signs for the constants is a determinant ingredient for the solution. Present results are showed for only demonstrative proposes.

4.3.2 Exact solution

An exact solution for the metric coefficients is obtained when $X = \lambda(t) a f(a)$. Substituting this ansatz in (4.20), this implies,

$$n = \frac{\dot{a}}{\lambda(t) f(a)}. \tag{4.35}$$

Now, we can integrate Eq. (4.27) to give,

$$b = \mathcal{C} f(a), \tag{4.36}$$

Finally, from (4.26) we obtain a differential equation for a

$$a' = \ \mathcal{C} f(a) \sqrt{\lambda(t)^2 f(a)^2 + k - \frac{\Lambda_5}{6} a^2 + \frac{C_{DR}}{a^2}}. \tag{4.37}$$

Usually, to have a stabilized bulk, b is taken to be the unity, but in general, $b = f(a) = \mathcal{C}^{-1}$ so, we can make, without lost of generality, $\mathcal{C} = 1$. In this work, $f(a)$ takes a general form $f(a) = a^{m/2}$, which is the case of non-static internal dimensions. Substituting last hypotesis in the equations (4.35), (4.36) and (4.37) immediately yields

$$
\begin{aligned}
n &= \frac{\dot{a}}{\lambda(t)a^{m/2}} && (4.38)\\
b &= a^{m/2}, && (4.39)\\
a' &= a^{m/2}\sqrt{\lambda^2 a^m + k - \frac{\Lambda_5}{6}a^2 + C_{DR}a^{-2}}, && (4.40)
\end{aligned}
$$

The term λ^2 behave as curvature, cosmological constant (CC) and dark radiation when $m = 0, 2, -2$ respectively. The general metric that solve the 5-D Enistein equations must satisfy

$$\int_{a_0}^{a} \frac{a^{1-m/2}}{\sqrt{\lambda^2 a^{m+2} - \frac{\Lambda_5}{6}a^4 + ka^2 + C_{DR}}} da = \pm y. \tag{4.41}$$

This integral is only solvable for $m = 0, 2, -2$. The case $m = 0$ is easy to solve and the general solution is shown in [44]. The cases m=2,-2 can be solving only with elliptic integrals.

- **For m=0** On this case, the integral solution can be reduced to

$$a^2 = A\cosh(\mu y) + B\sinh(\mu y) + C, \tag{4.42}$$

for negative cosmological constant $\Lambda_5 < 0$,

$$a^2 = A\cos(\mu y) + B\sin(\mu y) + C, \tag{4.43}$$

for positive cosmological constant, $\Lambda_5 > 0$, or finally

$$a^2 = \left(\lambda^2 + k\right) y^2 + Dy + E, \tag{4.44}$$

for nule cosmological constant, $\Lambda_5 = 0$.

- **For m=+2**,

$$F\left[sin^{-1}\hat{a}, \sqrt{R_-/R_+}\right] = \pm\sqrt{R_+(\lambda^2 - \Lambda_5/6)}\mathcal{C}y, \tag{4.45}$$

where

$$
\begin{aligned}
\hat{a} &= \frac{a}{\sqrt{R_-}}, \\
R_\pm &= \frac{-k \pm \sqrt{k^2 - 4C_{DR}(\lambda^2 - \Lambda_5/6)}}{2(\lambda^2 - \Lambda_5/6)}, \\
0 &< \frac{R_-}{R_+} < 1, \\
0 &< \lambda^2 - \frac{1}{6}\Lambda_5,
\end{aligned}
$$

- **For m=-2**

$$
\begin{aligned}
& F\left[sin^{-1}\hat{a}, \sqrt{R_-/R_+}\right] - E\left[sin^{-1}\hat{a}, \sqrt{R_-/R_+}\right] \\
&= \pm\sqrt{\frac{-\Lambda_5}{6R_+}}\mathfrak{C}y,
\end{aligned}
\tag{4.46}
$$

where

$$
\begin{aligned}
R_\pm &= \frac{-k \pm \sqrt{k^2 + 4\Lambda_5(C_{DR} + \lambda^2)/6}}{-2\Lambda_5/6}, \\
0 &< \frac{R_-}{R_+} < 1, \\
0 &< -\frac{1}{6}\Lambda_5,
\end{aligned}
$$

being $F[x,y]$ and $E[x,y]$ the incomplete elliptic integrals of the first and second kind respectively. The utility for this solutions, the corresponding graphics and its cosmological interpretation is not reported in this thesis.

- **Vacuum solutions** Flatness and non dark-radiation conditions are imposed, $k = C_{DR} = 0$; these last hypotheses are well justified due to the observation of an almost null curvature in the observable universe[4], and the fast decaying of bulk terms due to the scaling term a^4 for the dark radiation as well as the observable parameters imposed by nucleosynthesis[57]. So, Eq. (4.41) must be reduced to

$$
\frac{da}{a^{m/2}\sqrt{\lambda^2 a^m - \frac{\Lambda_5}{6}a^2}} = dy. \tag{4.47}
$$

For $\Lambda_5/6$ small enough, [1], a general solution of the metric coefficient a is

$$a(y) \approx a_0 \left[1 + (1-m)\lambda a_0^{m-1} y\right]^{1/(1-m)} \tag{4.48}$$

For $m \neq 1$. The case $m = 1$ is the usual exponential solution, $a = a_0 e^{\lambda y}$. Last result reveals that scale factor a evolves as a power of the fifth coordinate. If well there is not an important dependence of the Λ_5 term, it is not zero at all. If we want to be consistent with the reduction of hierarchy along the fifth dimension, we must impose $a_c\, /a_0 \approx [1 + (1-m)Ky_c]^{1/(1-m)} \approx 10^{-15}$, where $K \equiv \lambda a_0^{m-1}$. In the next chapter we will show K is proportional to the negative of energy density.

Ky_c	m
-1	0
-10^{15}	$+2$
$-1/3$	-2
$-1/2$	-1
-50	$+1$

Table 4.2 Values of the size of the fifth dimension necessary to reduce the hierarchy, in concordance with the RS method.

4.4 Metric coefficients and modified Friedmann equations

We have shown in a previous work that a general solution of the metric coefficients in a vacuum bulk is obtained by evaluating Eq. (4.48) at the boundary conditions (4.8), and identifying $\lambda a_0^{m-1} = -\frac{\kappa_{(5)}^2}{6}\rho_0 b_0$, to obtain

$$a(t, y) = a_0 \left[1 + (m-1)\frac{\kappa_{(5)}^2}{6}\rho_0 b_0 y\right]^{(1-m)^{-1}}, \tag{4.49}$$

[1]The term $-\Lambda_5/6$ is subdominant in the very early times regime, when $a \to 0$, only for $m \leq 2.$, it can be shown from Eq. (4.47)

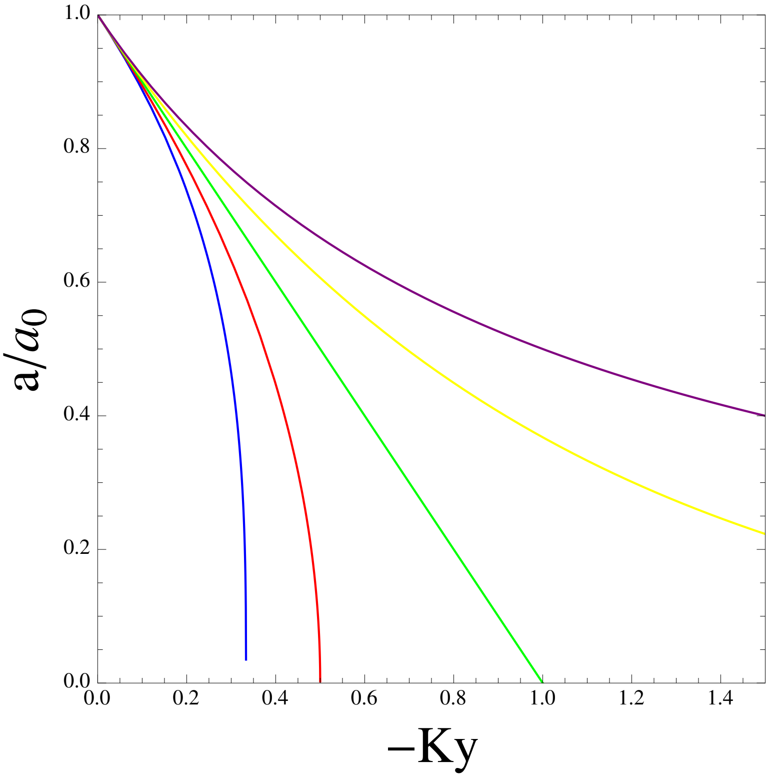

Figure 4.2 Plot of the reduction of the hierarchy for the metric coefficient $a(y)/a_0$ according with Ec. (4.48). $m = -2$, blue; $m = -1$, red; $m = 0$, green; $m = 1$, yellow and $m = 2$ purple.

The coefficients n and b are easily calculated by using (4.38) and (4.39) with aid of the boundary conditions (4.9) and the conservation equation (4.7),

$$n(t,y) = n_0 \left[1 + \left(\frac{m}{2} + 2 + 3\omega_0\right)\frac{\kappa_{(5)}^2}{6}\rho_0 b_0 y\right]\left[1 + (m-1)\frac{\kappa_{(5)}^2}{6}\rho_0 b_0 y\right]^{-\frac{m}{2m-2}} \tag{4.50}$$

$$b(t,y) = b_0 \left[1 + (m-1)\frac{\kappa_{(5)}^2}{6}\rho_0 b_0 y\right]^{-\frac{m}{2m-2}} . \tag{4.51}$$

The functions a_0, n_0 and b_0 correspond to the time-dependent values of the metric coefficients in the brane at $y = 0$, and m is a parameter which determinate the bulk geometry.

Now, we will proceed to calculate an equation that relates the evolution of the energy density in both branes. With the hypothesis $k = C_{DR} = 0$, Equation (4.26) can be written as

$$X^2 = Y^2 + a^4\frac{\Lambda_5}{6} \tag{4.52}$$

and using the definitions (4.20), $X = a\dot{a}/n = \lambda a^{1+m/2}$, $Y = aa'/b$,

$$\lambda^2 a^{m-2} = \left(\frac{a'}{ab}\right)^2 + \frac{\Lambda_5}{6} \tag{4.53}$$

Now, evaluating the above equation in the boundary of the two branes and comparing,

$$\left(\frac{\kappa_{(5)}^2}{6}\rho_c\right)^2 + \frac{\Lambda_5}{6} = \left(\frac{a_c}{a_0}\right)^{m-2}\left[\left(-\frac{\kappa_{(5)}^2}{6}\rho_0\right)^2 + \frac{\Lambda_5}{6}\right] \tag{4.54}$$

Finally, using Eq. (4.49), we obtain the relationship among the evolution of energy densities in the two branes, ρ_c and ρ_0, given by

$$\left(\frac{\kappa_{(5)}^2}{6}\rho_c\right)^2 + \frac{\Lambda_5}{6} = \left[\left(-\frac{\kappa_{(5)}^2}{6}\rho_0\right)^2 + \frac{\Lambda_5}{6}\right]\left[1 + (m-1)\frac{\kappa_{(5)}^2}{6}\rho_0 b_0 y_c\right]^{\frac{m-2}{1-m}} , \tag{4.55}$$

Assuming a Friedman-Lemaitre-Robertson-Walker (FLRW) metric on the visible brane $y = y_c$, with $n_c = 1$, another useful equation can be obtained from 4.9, establishing the relationship between ω_0 and ω_c as

$$\omega_c = \frac{\omega_0 + (\frac{m}{6} - 1)(\frac{m}{2} + 2 + 3\omega_0)\frac{\kappa_{(5)}^2}{6}\rho_0 b_0 y_c}{1 + (\frac{m}{2} + 2 + 3\omega_0)\frac{\kappa_{(5)}^2}{6}\rho_0 b_0 y_c} \tag{4.56}$$

Now, we can evaluate $H = \dot{a}/a$ into Eq. (4.52),

$$H = \left(\frac{\dot{a}}{a}\right) = n^2 \left[\left(\frac{a'}{ab}\right)^2 + \frac{\Lambda_5}{6}\right], \tag{4.57}$$

to find the corresponding Hubble parameters in each brane, namely [57, 82]

$$\begin{aligned} H_0^2 &= n_0^2(\kappa_{(5)}^4 \rho_0^2/36 + \Lambda_5/6) && (4.58)\\ H_c^2 &= n_c^2(\kappa_{(5)}^4 \rho_c^2/36 + \Lambda_5/6). && (4.59) \end{aligned}$$

Using Eq. (4.50) we finally obtain

$$\begin{aligned} H_0^2 &= \left[\frac{\kappa_{(5)}^4}{36}\rho_{0,m}\left(\rho_{0,m} + 2\lambda_0\right) + \Lambda_4\right] \frac{\left[1 + (m-1)\frac{\kappa_{(5)}^2}{6}(\rho_{0,m} + \lambda_0)b_0 y_c\right]^{-\frac{m}{1-m}}}{\left[1 + \left(\frac{m}{2} + 2 + 3\omega_0\right)\frac{\kappa_{(5)}^2}{6}(\rho_{0,m} + \lambda_0)b_0 y_c\right]^2} \\ H_c^2 &= \left[\frac{\kappa_{(5)}^4}{36}\rho_{0,m}\left(\rho_{0,m} + 2\lambda_0\right) + \Lambda_4\right]\left[1 + (m-1)\frac{\kappa_{(5)}^2}{6}(\rho_{0,m} + \lambda_0)b_0 y_c\right]^{\frac{m-2}{1-m}} \end{aligned} \tag{4.60}$$

for all $m \neq 1$. In the above equations we have also assumed that the energy density and pressure in the visible brane can be written in the form $\rho_0 = \rho_{0,m} + \lambda_0$ and $p_0 = p_{0,m} - \lambda_0$[33]. Also, the term $\Lambda_4 = \kappa_{(5)}^4 \lambda_0^2/36 + \Lambda_5/6$ is taken as the effective 4dim CC of the observable universe and at the rest of the thesis I will neglect its contribution to the cosmology, i.e. $\Lambda_4 = 0$.

4.5 Brane dominated by a single component

A first treatment of the cosmological solutions in present model, is to assume the visible brane is dominated by a single component, with negligible tension ($\lambda_0 = 0$) such that ω_0 is a constant. For that reason, it is easy deduce, from energy conservation in hidden brane, $\rho_0 = \Gamma a_0^{-3(1+\omega_0)}$, where Γ can be chosen such that $\Gamma\frac{\kappa_{(5)}^2}{6} y_c = 1$. Therefore, the modified Friedmann equation and equation of state in the visible brane are

$$y_c^2 H_c^2 = a_0^{-6(1+\omega_0)}\left[1 + (m-1)a_0^{-3(1+\omega_0)+m/2}\right]^{\frac{m-2}{1-m}} \tag{4.61}$$

$$\omega_c = \frac{\omega_0 + (\frac{m}{6} - 1)(\frac{m}{2} + 2 + 3\omega_0)a_0^{-3(1+\omega_0)+m/2}}{1 + (\frac{m}{2} + 2 + 3\omega_0)a_0^{-3(1+\omega_0)+m/2}} \tag{4.62}$$

On the other hand, we can express a_c in terms of a_0 by means of 4.49 as

$$a_c = a_0 \left[1 + (m-1)a_0^{-3(1+\omega_0)+m/2}\right]^{\frac{1}{1-m}} \tag{4.63}$$

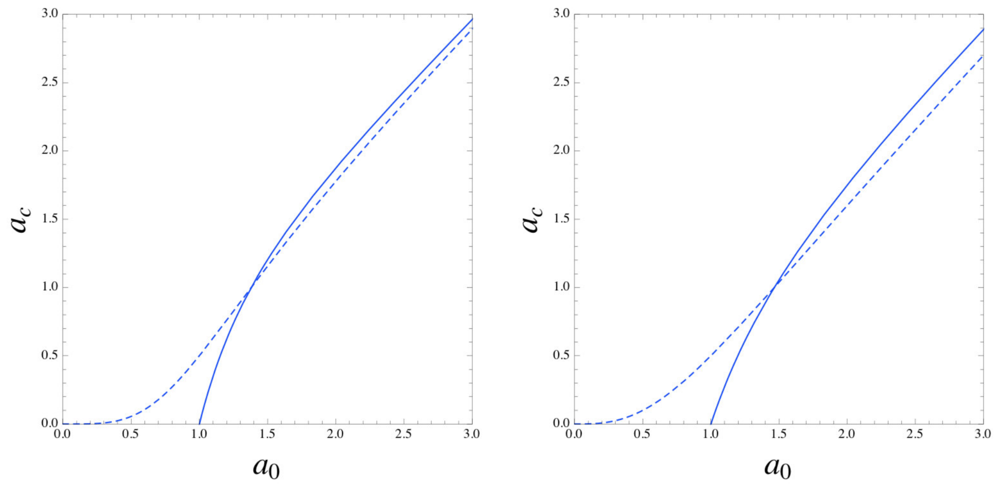

Figure 4.3 Plot of a_c as a function of a_0, for $\omega_0 = 1/3$ (left side) and $\omega_0 = 0$ (right side). Solid/dashed lines correspond to $m = 0, 2$ respectively.

Figure 4.3 shows the behavior of the a_0 and a_c functions: when $m = 0$, the scale factor in the hidden brane apparently starts from a positive value, which implies there is not a singularity here, while in the $m = 2$ case, both branes start from zero.

Figures 4.4 and 4.5 show the evolution of this system for $\omega_0 = 1/3$ and $\omega_0 =$ respectively. Dashed line represent the case $m = 2$ while solid line is the case for $m = 0$.

As a result of the analysis, we observe in Figure 4.4 that the parameter ω_c is not constant in time, but evolves departing from a period of dark energy domination to a radiation era. This is the main result of this section and the next step is consider a scalar field in the hidden sector such that inflation effects can be obtained in visible brane.

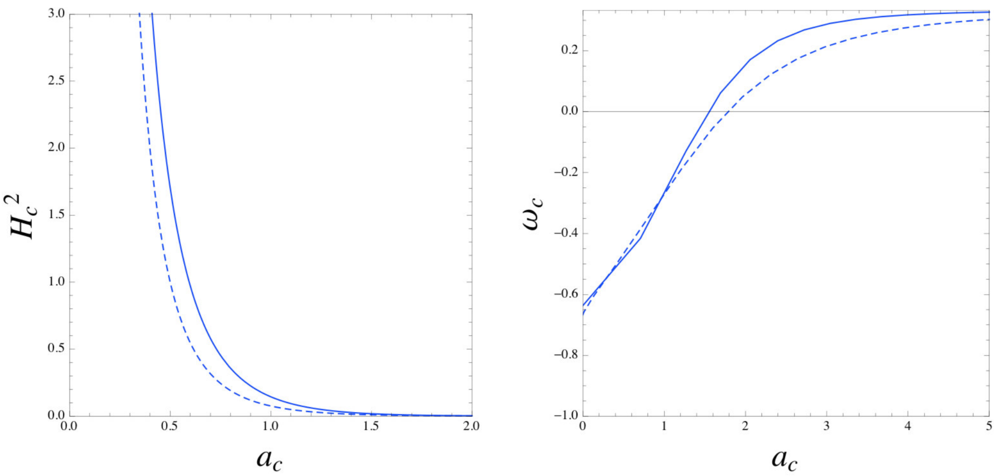

Figure 4.4 Plot of H_c and ω_c as a function of a_c for $\omega_0 = 1/3$. Note that the parameter of the equation of state in the hidden brane is initially $\omega_c = -2/3$ and eventually evolves towards the value 0 and subsequently 1/3. Solid/dashed lines correspond to $m = 0, 2$ respectively.

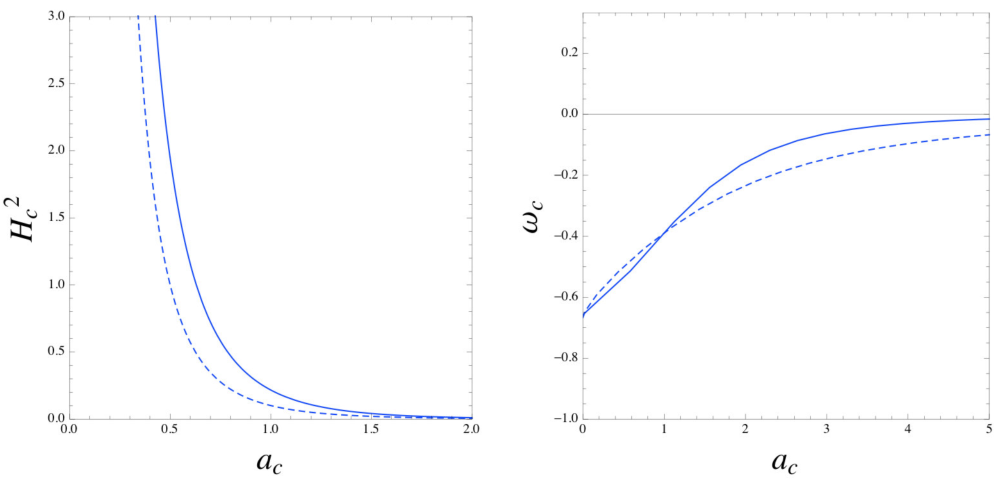

Figure 4.5 Plot of H_c and ω_c as a function of a_c for $\omega_0 = 0$. Note that the parameter of the equation of state in the hidden brane is initially $\omega_c = -2/3$ and eventually reaches the value 0. Solid/dashed lines correspond to $m = 0, 2$ respectively.

4.6 Scalar field living in the brane

In 1-brane RS-type brane-worlds, where the bulk has only a vacuum energy, inflation on the brane must be driven by a 4D scalar field trapped on the brane. In more general brane-worlds, where the bulk contains a 5D scalar field, it is possible that the 5D field induces inflation on the brane via its effective projection. More exotic possibilities arise from the interaction between two branes, including possible collision, which is mediated by a 5D scalar field and which can induce either inflation or a hot big-bang radiation era, as in the ekpyrotic or cyclic scenario or in colliding bubble scenario.

In this section, we study the case in which the visible brane is populated with a standard scalar field ϕ endowed with a scalar potential of the form $V(\phi) = m_\phi^2\phi^2/2$, where m_ϕ is the mass parameter of the scalar field, and with it the dynamics induced upon the visible brane through the topology considerations shown in Eqs. (4.4). We will establish the conditions that lead to inflation in both, visible and hidden, branes and the possible physical consequences of it.

To close the system of equations of motion, we must take into account the conservation equation of the scalar field in the hidden brane,

$$\ddot{\phi} + 3H_0\dot{\phi} + m_\phi^2\phi = 0\,. \tag{4.64}$$

Also, the energy density and pressure for the scalar field is defined by: $\rho_{0,m} = (1/2)(\dot{\phi}^2 + m_\phi^2\phi^2)$ and $p_{0,m} = (1/2)(\dot{\phi}^2 - m_\phi^2\phi^2)$. From here and with the scalar-field, it is possible to study the brane dynamics in two main cases: when radion effects are negligible and when radion play an important role in the dynamic.

4.6.1 Negligible radion in Brane Dynamics.

We start with the scenario in which the radion contribution is negligible, that is $\kappa_{(5)}^2\rho_0 b_0 y_c = \kappa_{(5)}^2\rho_0 R \ll 1$. Physically, this limit corresponds to an epoch in which the Hubble radius is much larger than radius of compactification, namely $H_0^{-1} \gg R$. Hence, Eqs. (4.60) can be written as

$$H_0^2 = H_c^2 = \frac{\kappa_{(4)}^2}{6}\frac{\rho_{0,m}^2}{\lambda_0} = \frac{\kappa_{(5)}^4}{36}\rho_{0,m}^2. \tag{4.65}$$

Because we are dealing with a scalar field, it is convenient to rewrite the equations of motion in terms of the new dimensionless variables:

$$x^2 \equiv \frac{\kappa_{(5)}^2}{12H_0}\dot{\phi}^2\,, \quad y^2 \equiv \frac{\kappa_{(5)}^2 m_\phi^2}{12H_0}\phi^2\,, \quad s \equiv \frac{m_\phi}{H_0}\,, \tag{4.66}$$

and then, the equations of motion are now

$$x' = 3x^3 - 3x - sy, \quad y' = 3x^2 y + sx, \quad s' = 6sx^2. \tag{4.67}$$

Here a prime indicates derivative with respect to the e-foldings number $N_0 \equiv \ln(a_0)$ and should not be confused with d/dy from the last section. However, variables x and y are subjected to the Friedman constraint $x^2 + y^2 = 1$, and then it is appropriate to consider one more change of variables[86]: $x = \cos\theta$, $y = \sin\theta$. Then, the equations of motion (4.67) can be reduced to the two following dimensional system

$$\theta' = 3\cos\theta\sin\theta + s\,, \quad s' = 6s\cos^2\theta\,. \tag{4.68}$$

The critical points, together with their stability, of this dynamical system are listed in Table 4.3. The stability was determined by calculating the eigenvalues and eigenvectors of the Jacobian matrix

$$\mathcal{M}_{(\theta,s)} = \begin{bmatrix} 3\cos(2\theta) & 1 \\ -6s\ \sin(2\theta) & 6\cos^2\theta \end{bmatrix}. \tag{4.69}$$

Critical point $\{\theta, s\}$, $n \in \mathbb{Z}$	Eigenvalue	Stability
$\{n\pi, 0\}$	$\{6, 3\}$	Unstable
$\{(n+\frac{1}{2})\pi, 0\}$	$\{-3, 0\}$	Saddle

Table 4.3 Properties of the critical points of the dynamical system (4.68). All critical points are unstable, but the saddle points are the source for inflationary solutions.

Typical solutions of the dynamical system on the (θ, s) plane are shown in Fig. 4.6. We observe that the points $\{\theta = n\pi, s = 0\}$ (where the kinetic energy is dominant) are unstable. However, it is possible to see that the system eventually evolves towards the critical point $\{\theta = \pi/2, s \neq 0\}$ (where the potential energy is dominant). The existence of an inflationary solution is guaranteed by the condition $\ddot{a}_0/a_0 > 0$. So, is straightforward to show

$$\omega_0 = \frac{x^2 - y^2}{x^2 + y^2} = \cos(2\theta) < -2/3, \tag{4.70}$$

which implies that inflation occurs only in regions in the vicinity of $\pi/2$, more precisely,

$$\theta \in \left(arccos\sqrt{1/6}\ ,\ \pi - arccos\sqrt{1/6} \right). \tag{4.71}$$

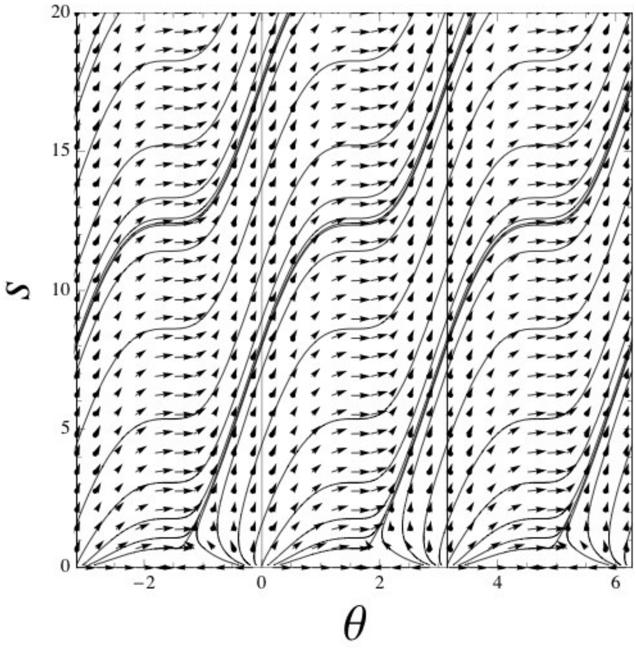

Figure 4.6 Numerical solutions of the dynamical system (4.68) on the $\theta - s$ plane. Note that the points $\{n\pi, 0\}$ are unstable, whereas the points $\{(n+\frac{1}{2})\pi, 0\}$ are saddle. We can see that the solutions rapidly evolve towards a potential dominated epoch, but all eventually end up in an oscillatory regime at late times.

Slow-roll conditions $\epsilon, |\eta| \ll 1$ are supposed sufficient to obtain inflation, and we can restrict once more the dynamical variables:

$$\epsilon = -\frac{\dot{H}_0}{H_0^2} = 6x^2 = 6\cos^2\theta \ll 1\,, \tag{4.72}$$

$$\eta = \left|\frac{\ddot{\phi}}{H_0\dot{\phi}}\right| = \left|3 + s\frac{y}{x}\right| = |3 + s\tan\theta| \ll 1\,, \tag{4.73}$$

Inflation ends when $\epsilon = 6\cos^2\theta_{end} = 1$, so $\theta_{end} = n\pi \pm \arccos\sqrt{1/6}$. Subsituting in $\eta = 1$, we have $s_{end} = -3/\tan\theta_{end} = 1.34164$. Therefore, we expect inflation occurs while $s \ll 1.34$ and $\cos^2\theta \ll 1/6$ (See Fig. 4.7).

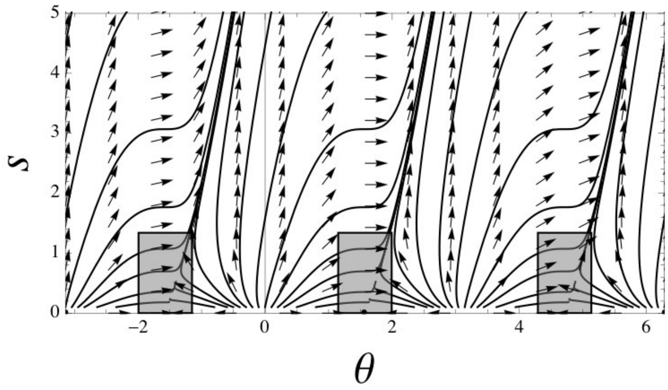

Figure 4.7 Shadow regions limits the possible values for θ and s which satisfies $\epsilon,\ \eta < 1$. Inflation occurs only in shaded region implying that only some particular conditions generates a sufficient and well behaved inflation. See the text for more details.

The above equations restrict the variables θ and s to be $\theta \approx \pi/2$, $s \approx 0$. For sufficiently small initial values of variable s, the solutions move closely to the saddle points, for which exists a natural exponential expansion in our (visible) brane that satisfies the conditions for inflation[86]. Figures 4.9 and 4.10 show the inflationary solution in the hidden brane when $\theta_i = \pi/2$ and $s_i = 0.1025$. It is possible to observe that quadratic SF (in a radion negligible limit), still be a good candidate to inflation due to the topological effects generated by branes, giving a new rich dynamic that extend its properties and the capability to obtain a well behaved inflation, which could be in concordance with observations.

It is important to remark, that the last ideas must be studied taking in consideration the observational probes of inflation and comparing with this. However, the contrast with observations is not relevant for this thesis and it will be subject of further works.

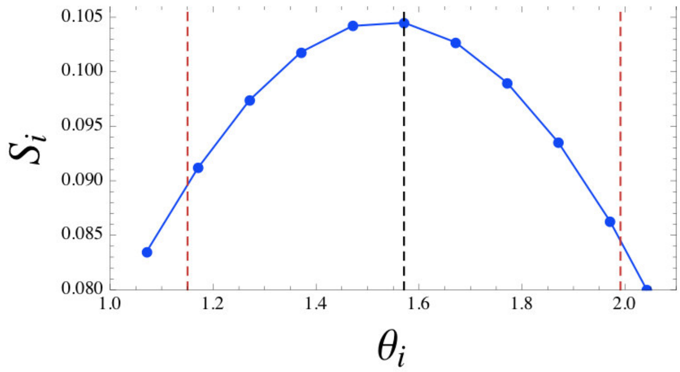

Figure 4.8 Numerical simulation of the initial conditions (s_i, θ_i) for the system (4.68) which guarantee an exponential expansion during $N_0 = 70$ e-folds. Note, when $\theta_i \approx \frac{\pi}{2}$, $s_i \approx 10^{-1}$, i. e. $m_\phi \approx 10^{-1} H_{0i}$. Red dashed lines limits the interval $\left(\cos^{-1}\sqrt{1/6}\ ,\ \pi - \cos^{-1}\sqrt{1/6}\right)$ within which an exponential expansion occurs.

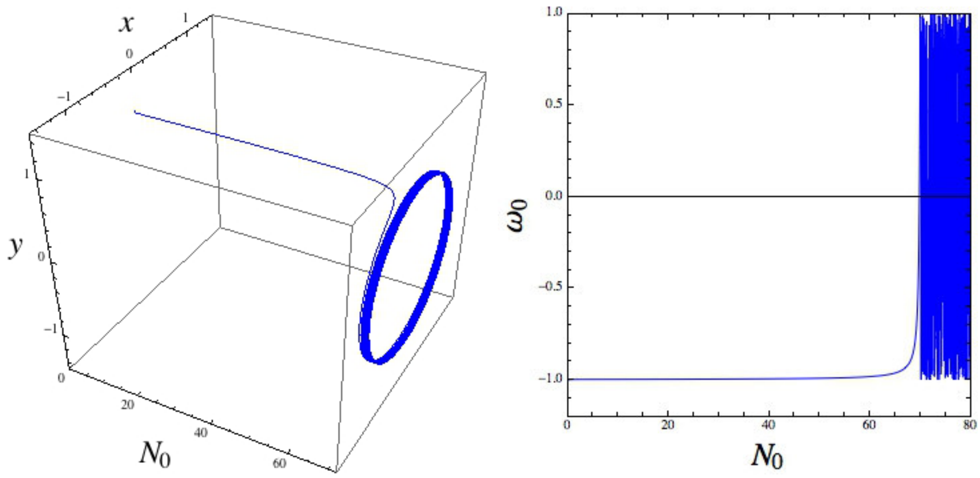

Figure 4.9 (Top) Numerical solutions for the SF variables x and y form Eq. (4.67) (Bottom) for the effective equation of state, with initial values $\theta_i = \pi/2$ and $s_i = 0.1025$. This values guarantee that the universe expands inflationarily for about $N_0 \approx 70$ e$-$foldings before oscillating around the minimum of potential.

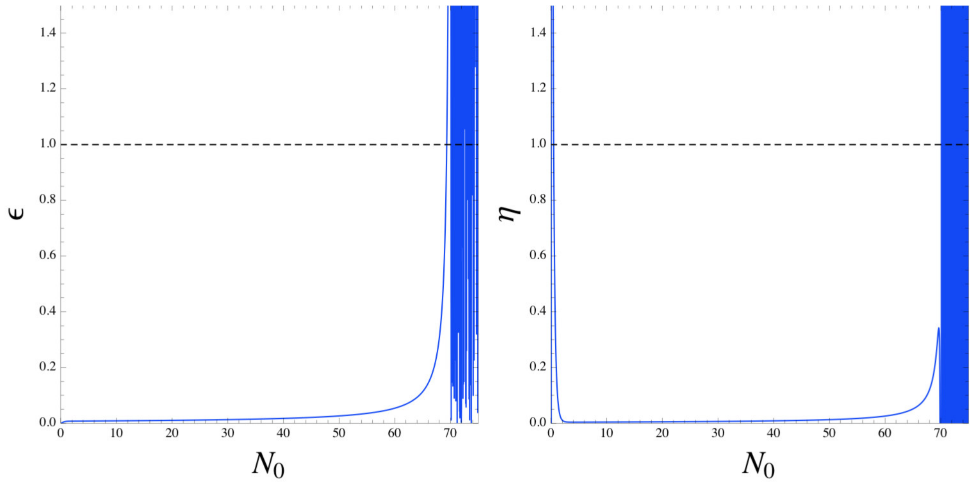

Figure 4.10 Plot of the slow-roll conditions for the system (4.67). Inflation is maintained during $N_0 \approx 70$ e-foldings approximately to obtain the characteristics of the observable universe. After this epoch starts the oscillatory behavior caused by the scalar field potential and could be associated to the reheating epoch.

4.6.2 Effects of the radion in our visible brane

We now turn our attention to the full system of equations without neglecting the contribution of the radion terms. The special case $m = 0$ is considered newly and $\rho_{0,m} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, and then Eqs. (4.60) becomes

$$H_0^2 = \frac{\frac{\kappa_{(5)}^4}{36}\rho_{0,m}\left(\rho_{0,m} + 2\lambda_0\right)}{\left[1 + (2 + 3\omega_0)\,\frac{\kappa_{(5)}^2}{6}(\rho_{0,m} + \lambda_0)R\right]^2}\,, \tag{4.74}$$

$$H_c^2 = \frac{\frac{\kappa_{(5)}^4}{36}\rho_{0,m}\left(\rho_{0,m} + 2\lambda_0\right)}{\left[1 - \frac{\kappa_{(5)}^2}{6}(\rho_{0,m} + \lambda_0)R\right]^2}\,. \tag{4.75}$$

In order to write the above equations in a simplified form, we define the u, v and C dimensionless variables, such that

$$\frac{\kappa_{(5)}^2}{6}R\rho_{0,m} = \frac{\kappa_{(5)}^2}{6}R\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] = u^2 + v^2\,, \tag{4.76}$$

$$\frac{\kappa_{(5)}^2}{6}R\lambda_0 = C\,, \tag{4.77}$$

and therefore Eqs. (4.74) and (4.75) are rewritten as

$$RH_0 = \frac{\sqrt{u^2+v^2}\sqrt{u^2+v^2+2C}}{1+5u^2-v^2-C}\,, \tag{4.78}$$

$$RH_c = \frac{\sqrt{u^2+v^2}\sqrt{u^2+v^2+2C}}{1-(u^2+v^2+C)}\,, \tag{4.79}$$

where RH_0 and RH_c are the dimensionless Hubble parameters for both branes. Note, from Eq. (4.79), a positive expanding brane can be possible for $u^2+v^2+C<1$. The evolution in time for the energy components is written as

$$\frac{du}{d\tau} = -3u\frac{\sqrt{(u^2+v^2)(u^2+v^2+2C)}}{1+5u^2-v^2-C} - \mathcal{F}v\,, \tag{4.80}$$

$$\frac{dv}{d\tau} = \mathcal{F}u\,, \tag{4.81}$$

where $\tau = t/R$, and $\mathcal{F}$ is a function which depends on the shape of the potential.

$$\mathcal{F} = \sqrt{\frac{12R}{\kappa_{(5)}^2}}\left(\frac{dv}{d\phi}\right)\,. \tag{4.82}$$

As a generalization, we can consider the polynomial case $V(\phi) = \lambda_\phi m_p^{4-r}\phi^r$, where m_p is the Planck mass and λ_ϕ is a coupling constant for the SF. Then, $\mathcal{F} = Av^{1-2/r}$ with

$$A = \frac{r}{2}\sqrt{\frac{12R}{\kappa_{(5)}^2}}\left(\frac{\kappa_{(5)}^2}{6}R\lambda_\phi m_p^{4-r}\right)^{1/r}\,, \tag{4.83}$$

being a constant. Now, we turn our attention to study the case of a quadratic potential due to its simplicity and the fact that in later epochs may be considered as DM (see for example [47],[86],[65],[15],[63],[5],[64],[62])

4.6.3 Analysis for quadratic Potential.

An interesting case due to its fundamental properties, is the quadratic potential. This is because this potential reproduce the DM behavior and even inflation. For the quadratic potential case $r=2$, we have

$$A = \sqrt{2\lambda_\phi m_p^2}R = m_\phi R, \quad C = \sqrt{-\Lambda_5}R. \tag{4.84}$$

To ensure that SF reproduces an inflationary epoch, the following condition must be satisfied

$$
\begin{aligned}
R^2\frac{\ddot{a}_0}{a_0} &= \frac{(u^2+v^2)(u^2+v^2+2C)(1+35u^2-v^2-C)}{(1+5u^2-v^2-C)^3} \\
&+\frac{12Auv\sqrt{u^2+v^2+2C}-6u^2(u^2+v^2+C)}{(1+5u^2-v^2-C)^2} > 0. \qquad (4.85)
\end{aligned}
$$

From the definition of the slow-roll parameters

$$
\epsilon = -\frac{\dot{H}_0}{H_0^2}, \qquad \eta = \left|\frac{\ddot{\phi}}{H_0\dot{\phi}}\right|, \qquad (4.86)
$$

we obtain

$$
\begin{aligned}
\epsilon &= -\frac{30u^2}{(1+5u^2-v^2-C)^3} - \frac{12Auv}{\sqrt{(u^2+v^2)(u^2+v^2+2C)}} \\
&+\frac{6u^2(u^2+v^2+C)}{(u^2+v^2)(u^2+v^2+2C)} \ll 1\,, \qquad (4.87) \\
\eta &= \left|3+\frac{Av(1+5u^2-v^2-C)}{u\sqrt{(u^2+v^2)(u^2+v^2+2C)}}\right| \ll 1\,. \qquad (4.88)
\end{aligned}
$$

In terms of the e-folding parameter, $N_0 = \ln(a_0)$, we can use the following identity

$$
\frac{d}{d\tau} = R\frac{d}{dt} = RH_0\frac{d}{dN_0}, \qquad (4.89)
$$

to rewrite the system of Eqs. (4.80) and (4.81) in a more useful way:

$$
\begin{aligned}
\frac{du}{dN_0} &= -3u - \frac{Av(1+5u^2-v^2-C)}{\sqrt{(u^2+v^2)(u^2+v^2+2C)}}\,, \qquad (4.90) \\
\frac{dv}{dN_0} &= \frac{Au(1+5u^2-v^2-C)}{\sqrt{(u^2+v^2)(u^2+v^2+2C)}}\,. \qquad (4.91)
\end{aligned}
$$

The constants A and C are interrelated and its value depends on the number of e-foldings and the initial values of v_i and u_i. In the next, we consider only the situation $u_i = 0, v_i \neq 0$, due that other cases are problematic in obtain an oscillatory behavior for the SF which is the base to understand the scalar field dark matter (SFDM)

4.6.4 Imposing conditions for inflation

As a first approximation, consider $u^2 \ll v^2$, while the inflation occurs, and u is maintained constant such that $du/dN_0 \approx 0$, then integrating N_0 in the interval $[0, N_0]$ the Eqs. (4.90) and (4.91) leads to

$$A^2 = \frac{3}{2N_0}\left[\frac{(1+C)(v_i^2 - v_f^2)}{(1-v_i^2-C)(1-v_f^2-C)} + ln\left(\frac{1-v_i^2-C}{1-v_f^2-C}\right)\right]. \tag{4.92}$$

Physically we expect $v_i > v_f$, during inflation. So, $A \to A_{min}$ when $C \to 0$

$$A^2_{min} = \frac{3}{2N_0}\left[\frac{(v_i^2 - v_f^2)}{(1-v_i^2)(1-v_f^2)} + ln\left(\frac{1-v_i^2}{1-v_f^2}\right)\right]. \tag{4.93}$$

On the other hand, $C \to C_{max}$ for $A \to \infty$, namely: $C_{max} = 1 - v_i^2$. The constants A and C are bounded by $0 < C < C_{max}$ and $A_{min} < A < \infty$. Fig. 4.11 shows different curves in the (A,C) plane when inflation is maintained for $N_0 \approx 70$ e-folds and $v_f = 0$.

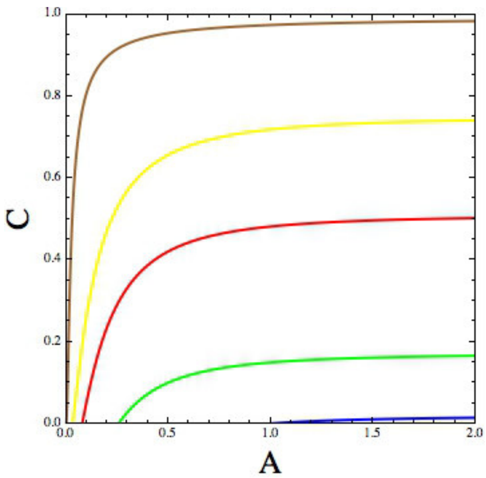

Figure 4.11 Plot of the A and C constants relationship from Eq. (4.92) with $N_0 \approx 70$ and $v_f = 0$. The colored lines {brown, yellow, red, green, blue} corresponds with $v_i = \{0.1, 0.5, 0.7, 0.9, 0.99\}$ respectively. Each one of the plotted point reproduces $N_0 \approx 70$ e−foldings, nevertheless only $C \ll A$ values gives an oscillatory scalar field when inflation ends.

The free parameters, for an initial time, are the constants C, u_i and v_i, under the restriction $u_i^2 + v_i^2 + C < 1$. For practical purposes, we are interested in values for $C \ll 1$. In the next Figs. 4.12, 4.13 and 4.14, we show the quadratic potential case with initial conditions $A = 1$, $C = 1.5 \times 10^{-3}$, $u_i = 0$ and $v_i = 0.91$. In this analysis we are interested in values for $C \ll 1$ to allow the system to oscillate around the minimum of the potential. The oscillations are necessary in models where reheating is required and with the aim of reproduce SFDM behavior [5, 15, 47, 62, 63, 64, 65, 86]. Eventually the terms u^2+v^2 become subdominant in comparison with C and Eq. (4.78) evolves in a standard-like form, but $\omega_0 \approx -1$, i.e. it behave as CC (see Fig. 4.12).

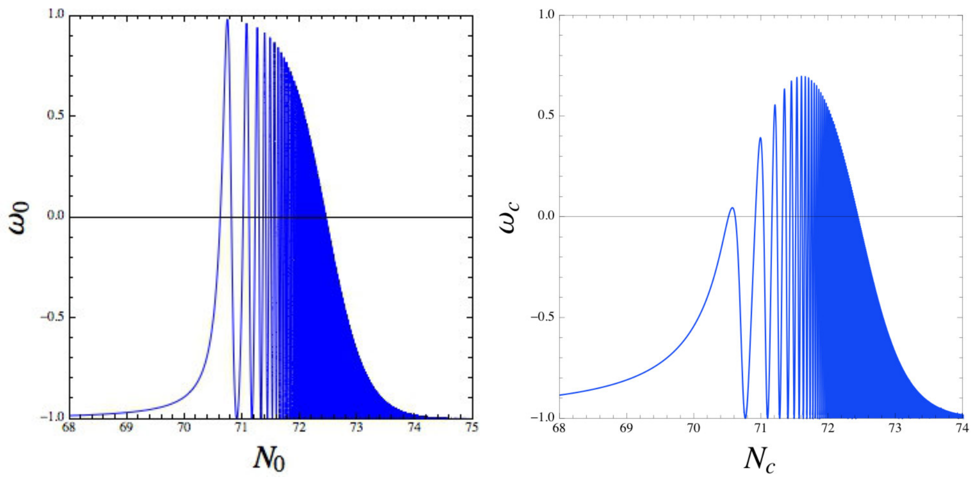

Figure 4.12 Plot of the ω_0 parameter. Here we can note the brane is dominated by a term with $\omega_0 = -1$, which is provided by the SF; note that when the system reach 70 e$-$folds of inflation, the scalar field oscillates. Eventually the SF contribution is subdominant compared with the C term and the parameter ω_0 newly behavest as a CC, but Eq. (4.78) ensures that the system enter in a standard-like evolution.

4.6.5 Inherited cosmology in visible brane

In previous sections, we have studied the cosmology on the hidden brane, but our interest is to recover a similar behavior in our brane, in which we expect to obtain inflation. In order to demonstrate these assertions, first, we must write

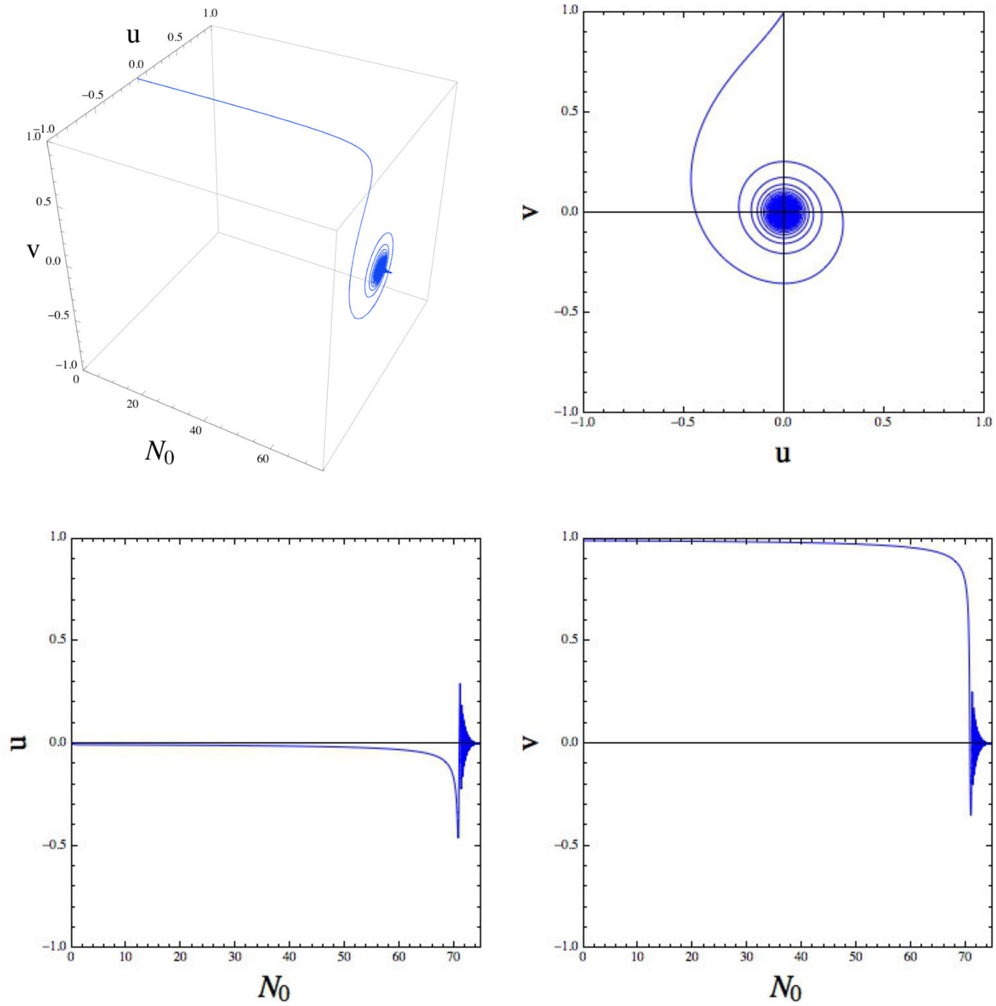

Figure 4.13 3D plot of the kinetic and potential terms for the SF. We can see the potential term is dominant until the system reach the 70 e−foldings. Finally the system oscillates around the minimum of potential.

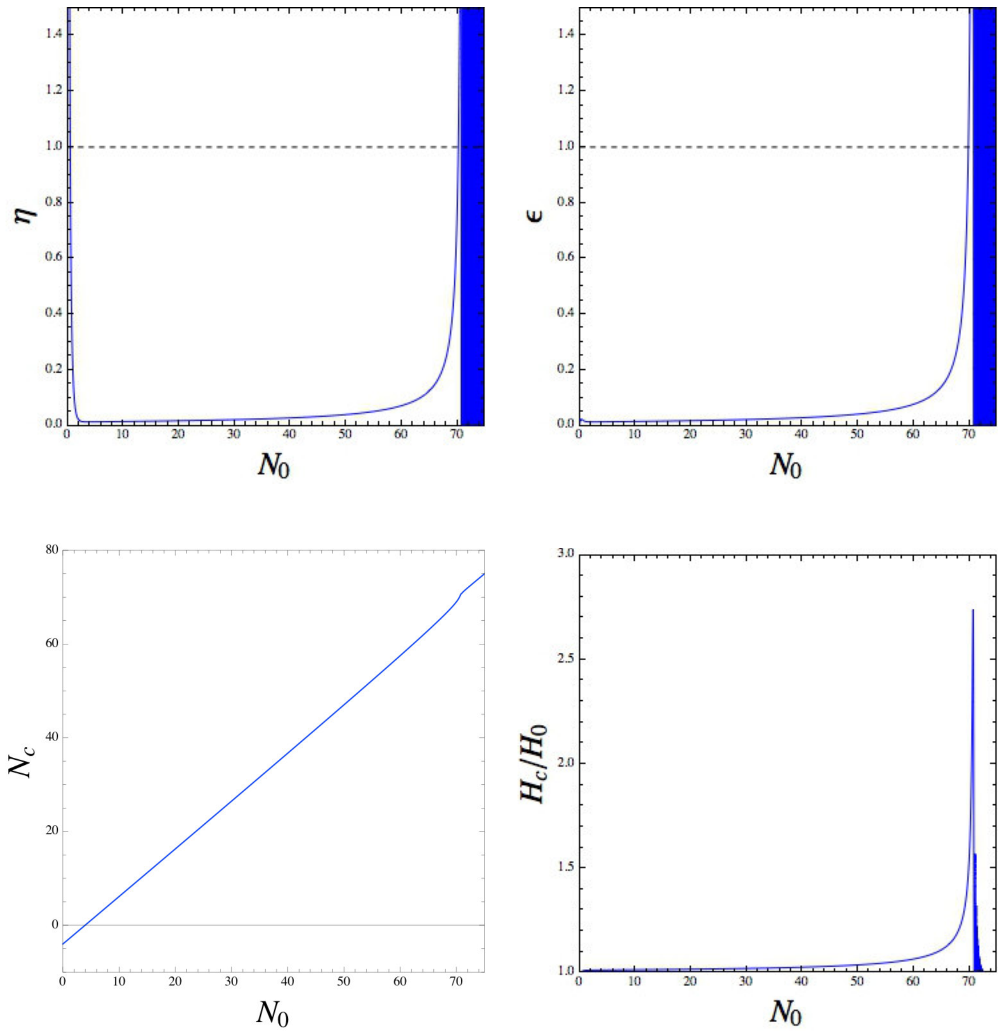

Figure 4.14 Plot of the inflationary conditions which are satisfied. We can note in this example H_c and H_0 are nearly of the same order and, as a consequence, the evolution is similar in both branes. The left plot shows the number of e-foldings in each brane is of the same order.

N_c as a function of N_0 using Eq. (4.49) for $m = 0$

$$a_c = a_0 \left[1 - \frac{\kappa^2_{(5)}}{6}\rho_0 R\right] = a_0 \left[1 - (u^2 + v^2 + C)\right] , \tag{4.94}$$

$$N_c = N_0 + \ln\left[1 - (u^2 + v^2 + C)\right] . \tag{4.95}$$

In a previous section, we have demonstrated ω_c is written as

$$\omega_c = \frac{\omega_0 + (\frac{m}{6} - 1)(\frac{m}{2} + 2 + 3\omega_0)\frac{\kappa^2_{(5)}}{6}\rho_0 R}{1 + (\frac{m}{2} + 2 + 3\omega_0)\frac{\kappa^2_{(5)}}{6}\rho_0 R}, \tag{4.96}$$

and written the above equation in our new variables, we obtain

$$\omega_c = \frac{\frac{u^2 - v^2 - C}{u^2 + v^2 + C} - 5u^2 + v^2 + C}{1 + 5u^2 - v^2 - C}. \tag{4.97}$$

Finally, we can easily plot Eqs. (4.95) and (4.97) by using N_0 as a parametric variable. It is seen in Fig. 4.12 the parameter $\omega_c = p_c/\rho_c$ behaves as cosmological constant during inflation. Later, when $N_c = 70$ e-folds are reached, the system oscillates as if it were dominated by a scalar field. The previous statements, demonstrate that the behavior in the hidden brane is inherited to the visible brane (our universe), obtaining the inflation epoch and the oscillatory behavior which is characteristic of this model.

5

Discussion

Using previous results, for which the branes are connected by imposing topological constraints and assuming a vacuum bulk, (with the aim of obtain an exact solution for the metric coefficients), we study the behavior of SF in the hidden brane and their visible effects in our brane at high energies. First, under the consideration that the radion effects are negligible, we analyze the general solutions of the equations of motion and in particular, the effects on inflation. Next we consider, further, the case where the radion contributes with the brane evolution, and the dynamical system is analyzed with the aim of study the inflation in this regime. In order to be most explicit with the conclusions, we punctuated the main results in the following way:

- The universe goes towards different epochs: the first one is dominated by the kinetic energy of the field, the second one is dominated by the potential energy, and the last one is oscillatory.

- The solutions that drive out the inflationary epoch, and that also satisfy the slow roll conditions, are located near the saddle critical points. This is always the case if $m_\phi \ll H_0$, which generates an exponential expansion that is typical of inflationary models.

- It is important to remark that it is possible to obtain an expanding solution in the visible brane, as long as the effective (or apparently) energy density r_c is negative. But, it is possible to argue that this is a consequence of the Z_2 symmetry, as is also the case of RS models.

- The long time expansion is recovered if we consider (as an example) $A = m_\phi R = 1$, $C \ll 1$, $u_i = 0$ and $v_i \approx 1$. The value A and C are crucial for understanding the transition from expanding to oscillatory ages, and if well many of functional combinations (A,C) was probed, we show only one of them.

- The effective parameter of EoS, ω_0 behaves initially as CC, driving the universe in an inflationary epoch, but when it finish, the SF oscillates around the minimum of potential. Later, when the SF is subdominant, the term C drives the brane into an standard-like evolution, while the ω_0 parameter behaves also as CC.

- A similar behavior is obtained in our visible brane, concluding that the effects of inflation are due to a scalar field in a hidden sector of the bulk unreachable for local experiments.

As we stated previously and for the results obtained during this work, we demonstrate that brane inflation is richer than ordinary cosmological inflation due to the extra terms that provide the topological configuration of the five and four dimensional structure and its mutual interaction. The brane scenario help us to alleviate many of the problems that suffers inflation and even, as we show here, suggest a unified model for inflation, DM and DE only varying the topological conditions. Despite of the extensive study, it is necessary to analyze the perturbed dynamic equations as well as the growth of structure caused by the primordial fluctuations during inflationary epoch, but this is not subject for the present thesis.

A thorough analysis of the dynamics of the universe, as well as the comparison with the recent cosmological observations, is required to test models that challenge the Λ-CDM and that try to change the current well stablished paradigms; however there are many models that attempt to solve problems in cosmology but they do it in an isolated manner and if well they solve some problems, they do not solve others. This thesis only focuses on building a model that provides a mechanism of interaction between two hypothetical branes, that although this is a toy model, it shows some of the implications of having an invisible brane in a region beyond our reach; although it has been shelved its utility to solve other important problems, such as the hierarchy problem, dark matter and dark energy of formation or structure, to name a few, which is sure to fail. Because one of the noblest purposes of toy models is to establish where theory is valid and where it is not, the work that has gone into them should not be discredited: one cannot solve everything in this lifetime.

Have multidimensional models been discarded today? While today are very undervalued brane models (which were very popular in the late 90's and early 2000's), there is nothing to tell us why the universe necessarily has only 4 dimensions, although there is no confirmation of any additional dimension. Despite the large amount of acceptance they have had in the scientific community, brane world models seem to reach a point where the lack of corroboration lead to its extinction, but perhaps these studies, including that presented in this thesis, mark a path to the conception of new fundamental theories under which in a few years, other problems bigger than the current ones will be solved

Brane models open a range of possibilities to explore new five dimensional phenomenology and although they arise new unknowns (such as stabilization of the radion, the nature of the 5-D fields, other hierarchies, etc) they provide relevant information on how the early universe could be if there is an extra dimension. The amount of models depends largely on the number of hypotheses (which is the case of this thesis, where exact solutions to Einstein 's equations from a particular ansatz are obtained), and new information can be obtained for each model in particular, so it sets a precedent for future work based on these models and research.

Declaration

I herewith declare that I have produced this thesis without the prohibited assistance of third parties and without making use of aids other than those specified; notions taken over directly or indirectly from other sources have been identified as such. This thesis has not previously been presented in identical or similar form to any other.

The thesis work was conducted under the supervision of Dr. Luis Ureña at División de Ciencias e Ingenierías Campus León, Universidad de Guanajuato

León Guanajuato, México
Febrero 2025

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Universidad de Guanajuato

CAMPUS LEÓN
DIVISIÓN DE CIENCIAS E INGENIERÍAS
DEPARTAMENTO DE FISICA

Asunto: Revisión de tesis de doctorado
León, Gto., Febrero de 2025

DR. MODESTO ANTONIO SOSA AQUINO
DIRECTOR DE LA DIVISIÓN DE CIENCIAS E INGENIERÍAS
PRESENTE

Por este medio le comunico que he revisado la tesis **Cosmological solutions for a two branes system in a generalized Randall-Sundrumm model**, escrita por el M. en F. **Juan Luis Pérez Pérez**, para efecto de presentarla para la obtención del grado de Doctor en Física en la División de Ciencias e Ingenierías de la Universidad de Guanajuato.

El texto de la tesis se encuentra completo y se presentan resultados interesantes sobre el estudio de modelos cosmológicos con branas para modelar la energía oscura del universo. En mi opinión, las hipótesis de trabajo y el análisis teórico se conectan bien con los resultados presentados, incluso dentro del contexto numérico. Igualmente, he podido ver que el texto fue modificado por el autor para reflejar las sugerencias y comentarios que le fueron expresados durante la revisión. En mi opinión la tesis cumple con los elementos necesarios para ser defendida ante el comité sinodal asignado en la fecha próxima que sea acordada de manera conjunta.

Agradeciendo su amable atención, aprovecho la ocasión para enviarle un cordial saludo.

ATENTAMENTE
"LA VERDAD OS HARÁ LIBRES"

L. A. U. L.

DR. LUIS ARTURO UREÑA LÓPEZ
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León, Gto, 29 de enero de 2025

Asunto: Carta liberación Juan Luis Pérez Pérez

Dr. Modesto Antonio Sosa Aquino
Director de la DCI

Como sinodal del estudiante de Doctorado en Física **Juan Luis Pérez Pérez**, me permito comentar que he leído el manuscrito de su tesis *Cosmological solutions for a two branes system in a generalized Randall-Sundrum model.*

Considero que el trabajo realizado está al nivel de un doctorado, por lo que le he enviado mis sugerencias y me permito recomendar que haga los trámites administrativos correspondientes para que se presente lo más pronto posible.

"LA VERDAD OS HARA LIBRES"

Dr. José Socorro García Díaz
Sinodal

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UNIVERSIDAD DE GUANAJUATO

22 de enero de 2025

Asunto: Carta de revisión de tesis de Juan Luis Pérez Pérez

Dr. Modesto Antonio Sosa Aquino
Director de la División de Ciencias e Ingenierías, Campus León
Universidad de Guanajuato
Presente

Estimado Dr. Modesto,

En mi calidad de integrante del comité sinodal (oficio número SAC-148/2024) del estudiante de Doctorado Juan Luis Pérez Pérez (NUA:), por este medio informo a usted que he revisado su tesis titulada "Cosmological solutions for a two branes system in a generalized Randall-Sundrum model" que realizó Juan Luis con el fin de obtener el grado de Doctor en Física.

El trabajo de Juan Luis posee el contenido y la relevancia necesaria como trabajo de investigación y Juan Luis ya ha considerado e implementado las correcciones sugeridas por un servidor. Considero que su trabajo de tesis está listo para ser defendido públicamente.

Sin más por el momento, me despido de usted con un cordial saludo.

Dr. José Luis López-Picón
Departamento de Física
División de Ciencias e Ingenierías, Campus León
Universidad de Guanajuato
email: jl_lopez@fisica.ugto.mx

UNIVERSIDAD DE GUANAJUATO
Campus León
División de Ciencias e Ingenierías

León, Gto., 5 de noviembre de 2024

Director Dr. Modesto A. Sosa Aquino

División de Ciencias e Ingenierías

PRESENTE

Estimado Dr. Sosa Aquino:

Por medio de este conducto le informo que he leído y discutido detalladamente el trabajo de tesis del **M. en F. Juan Luis Pérez Pérez** titulado "*Cosmological solutions for a two branes system in a generalized Randall-Sundrumm model*", dirigido por el Dr. Luis A. Ureña López, para obtener el grado de Doctor en Física. Estoy satisfecho con el contenido de la tesis, la contribución y el conocimiento obtenido y generado por Juan Luis, por lo que no tengo inconveniente porque el trabajo sea defendido en la fecha que resulte más conveniente.

Sin otro particular por el momento, aprovecho la ocasión para enviarle un cordial saludo y quedo atento a cualquier información requerida derivada del contenido de este documento.

Atentamente,

"La Verdad Os Hará Libres"

Dr. Ramón Castañeda Priego

Profesor Titular C

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UNIVERSIDAD DE GUANAJUATO

Asunto: Revisión de Tesis
León, Guanajuato, febrero de 2025

Dr. Modesto Antonio Sosa Aquino
Director
División de Ciencias e Ingenierías, Campus León
Universidad de Guanajuato
PRESENTE

Estimado Dr. Modesto Sosa:

Por medio de la presente le informo que he revisado la tesis "**Cosmological solutions for a two branes system in a generalized Randall-Sundrumm model**" escrita por el estudiante del Doctorado en Física de la DCI, Juan Luis Pérez Pérez. En mi opinión la tesis esta lista para ser presentada y estoy de acuerdo que se proceda al examen recepcional, una vez que se cumplan los procedimientos administrativos correspondientes.

Sin más por el momento, le envío saludos cordiales.

ATENTAMENTE
"LA VERDAD OS HARÁ LIBRES"

DRA. ARGELIA BERNAL BAUTISTA

División de Ciencias e Ingenierías
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León, Guanajuato, a 6 de febrero de 2025

Dr. Modesto Antonio Sosa Aquino,
Director de la División de Ciencias e Ingenierías,
Universidad de Guanajuato

Por medio de la presente deseo manifestar mi aprobación para que el trabajo presentado por el alumno **Juan Luis Pérez Pérez** sea aceptado como Tesis para obtener el título de **Doctor en Física.** El trabajo aborda el estudio de un modelos cosmológico que considera la existencia de una quinta dimensión dentro del modelo de Randall-Sundrumm. Particularmente, se analizan las consecuencias e influencia de diferentes escenarios cosmológicos en la brana oculta, sobre la brana visible. El intercambio de dinámica de los campos involucrados abre la posibilidad de interpretar observables cosmológicos como resultado de la existencia de una dimensión extra. En ese sentido, el trabajo presentado por el estudiante de posgrado Juan Luis Pérez Pérez, posee las características necesarias para ser aceptado como tesis de doctorado en Física.

Como miembro del comité sinodal he estado en contacto con Juan, por lo que me he asegurado que sus conocimientos y capacidades en la investigación están bien fundamentados. Por todo lo anterior, doy mi aprobación para que la tesis titulada "*Cosmological solutions for a two branes system in a generalized Randall-Sundrumm model"* sea considerada para continuar con el proceso de titulación.

Sin más por el momento, aprovecho para enviarle un cordial saludo.

Atentamente,

Dr. Oscar Gerardo Loaiza Brito
Departamento de Física,
División de Ciencias e Ingeniería,
Campus León.
Ext: 8459

c.c.p. archivo